**8.1 Indeterminate Forms of Type 0/0**

| Ex 1: Evaluate this limit. $\lim_{x\to 0}\frac{e^x-1}{e^{-x}-1}$ | L'Hopital's Rule<br>If $\lim_{x\to u} f(x)=0$ and $\lim_{x\to u} g(x)=0$, then $\lim_{x\to u}\frac{f(x)}{g(x)}=\lim_{x\to u}\frac{f'(x)}{g'(x)}$ . |
|---|---|

Ex 2: Compute the following limits.

| (a) $\lim_{x\to 0^-}\frac{3\sin x}{\sqrt{-x}}$ | (b) $\lim_{x\to 0}\frac{\tan x - x}{\arcsin x - x}$ |
|---|---|

8.1 (continued)

Ex 3: Compute the following limits.

| | |
|---|---|
| (a) $\lim\limits_{x \to 4} \dfrac{x-4}{e^{2x-8}-1}$ | (b) $\lim\limits_{x \to 0} \dfrac{\cos x - 1 + \frac{1}{2}x^2}{x^4}$ |
| (c) $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{x^2+1}{\cos x}$ | (d) $\lim\limits_{x \to 0} \dfrac{x^2 \sin\left(\frac{1}{x}\right)}{\tan x}$ |

**8.2 Other Indeterminate Forms**

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|---|---|
| Ex 1: Evaluate the following limits.<br>(a) $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$ | <u>L'Hopital's Rule</u><br><br>If $\lim_{x\to u} f(x)=\pm\infty$ and $\lim_{x\to u} g(x)=\pm\infty$, then $\lim_{x\to u}\frac{f(x)}{g(x)}=\lim_{x\to u}\frac{f'(x)}{g'(x)}$ . |
| (b) $\lim_{x\to 1^+}\left(\frac{1}{x-1}-\frac{x}{\ln x}\right)$ | <u>Indeterminate Limit Forms</u>:<br><br>$\frac{0}{0}$ , $\frac{\pm\infty}{\pm\infty}$ , $0\cdot\infty$ , $\infty-\infty$ , $0^0$ , $\infty^0$ , $1^\infty$ cases<br><br>All of these cases are "competing."<br><br><u>Non-indeterminate Forms</u>:<br>(there's no competition going on here)<br><br>$\frac{0}{\infty}\to 0$<br>$\frac{\infty}{0}\to\infty$<br>$\infty+\infty\to\infty$<br>$\infty\cdot\infty\to\infty$<br>$0^\infty\to 0$<br>$\infty^\infty\to\infty$<br>$1^0\to 1$ |

8.2 (continued)

Ex 2: Compute the following limits.

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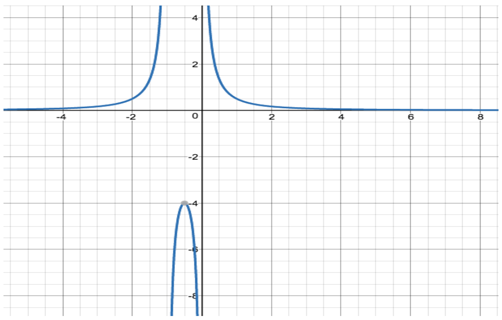
|---|---|
| (a) $\lim_{x \to 0^+} \left( \frac{1}{5}(2^x) + \frac{4}{5}(5^x) \right)^{\frac{1}{x}}$ | (b) $\lim_{x \to \infty} \left( \ln(x+2) - \ln(x-3) \right)$ |
| (c) $\lim_{x \to \pi} \left( \cot x \cdot \ln(\cos x) \right)$ | (d) $\lim_{x \to 0^+} \left( (x^x)^x \right)^x$ |

## 8.3 Improper Integrals

Ex 1: Evaluate the integral.




$$\int_{1}^{\infty} \frac{1}{x^2+x}\,dx$$

(1) $\displaystyle\int_{-\infty}^{b} f(x)\,dx = \lim_{a\to-\infty} \int_{a}^{b} f(x)\,dx$

(2) $\displaystyle\int_{a}^{\infty} f(x)\,dx = \lim_{b\to\infty} \int_{a}^{b} f(x)\,dx$

(3) $\displaystyle\int_{-\infty}^{\infty} f(x)\,dx = \lim_{a\to-\infty} \int_{a}^{0} f(x)\,dx + \lim_{b\to\infty} \int_{0}^{b} f(x)\,dx$

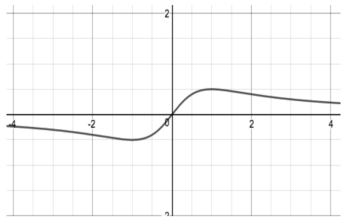
Note: If these limits, for (1) and (2) cases, evaluate to be a finite value, then the integral is said to converge and the answer is that finite value.

If these limits, for (1) and (2) cases, either do not exist or go to some sort of infinity, then the integral is said to diverge.

For case (3), if one of those integrals (on the right) diverges, then the entire integral diverges. That is, (3) converges to a finite value only if both pieces converge.

Ex 2: Evaluate the following integrals.

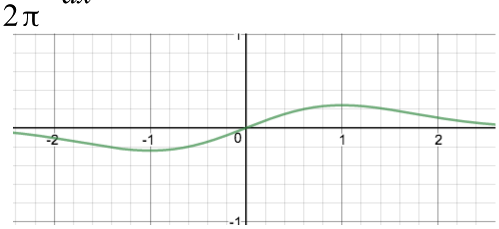
(a) $\displaystyle\int_{10}^{\infty} \frac{x}{x^2+1}\,dx$



(b) $\displaystyle\int_{-\infty}^{-1} \frac{1}{x^3}\,dx$

(c) $\displaystyle\int_{-\infty}^{\infty} \frac{2x}{\sqrt{x^2+25}}\,dx$

(d) $\displaystyle\int_{-\infty}^{\infty} \frac{x\,e^{-x^2/2}}{\sqrt{2\pi}}\,dx$



**8.4 Improper Integrals: Infinite Integrals**

<table>
<tr>
<td>Ex 1: Evaluate the following integrals.<br>(a) $\int_{\sqrt{5}}^{\sqrt{8}} \frac{x}{(16-2x^2)^{2/3}}\,dx$</td>
<td rowspan="2">(1) If there is a VA (vertical asymptote) at x = b, i.e. $\lim_{x\to b^-} f(x)=\pm\infty$ , and $f(x)$ is continuous on [a,b), then $\int_a^b f(x)\,dx=\lim_{m\to b^-}\int_a^m f(x)\,dx$ .<br>If this limit goes to infinity or DNE, then the integral diverges.<br><br>(2) If there is a VA (vertical asymptote) at x = a, i.e. $\lim_{x\to a^+} f(x)=\pm\infty$ , and $f(x)$ is continuous on (a,b], then $\int_a^b f(x)\,dx=\lim_{m\to a^+}\int_m^b f(x)\,dx$ .<br>If this limit goes to infinity or DNE, then the integral diverges.<br><br>(3) If there is a VA (vertical asymptote) at x = c, and $c \in (a,b)$ then<br>$\int_a^b f(x)\,dx=\lim_{m\to c^-}\int_a^m f(x)\,dx+\lim_{p\to c^+}\int_p^b f(x)\,dx$ .</td>
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<td>(b) $\int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{(\tan x-1)^2}\,dx$</td>
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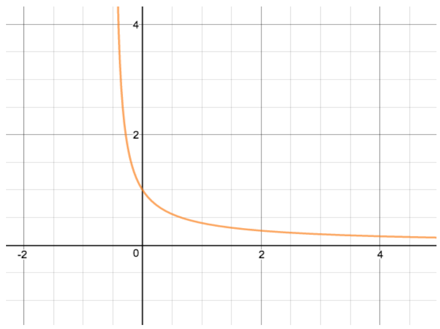
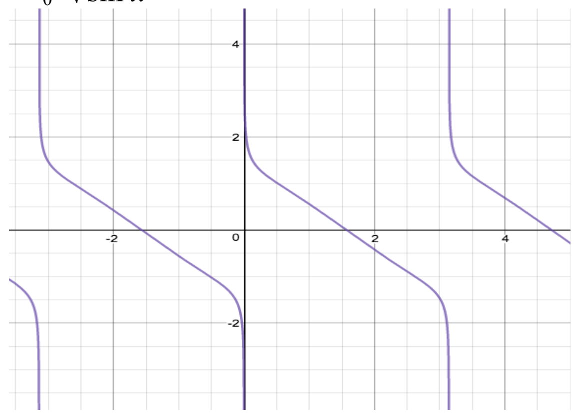
8.4 (continued)

Ex 2: Evaluate the following integrals.

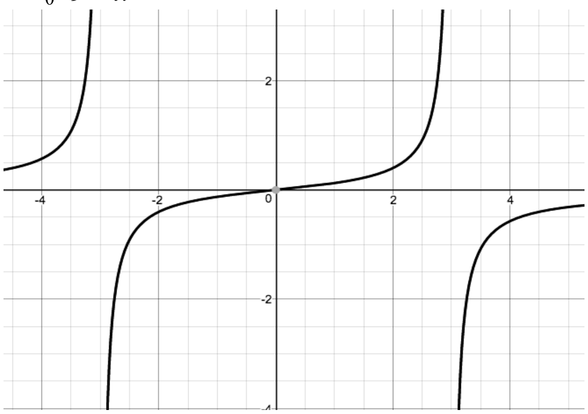
(a) $\displaystyle\int_{-1/2}^{1} (2x+1)^{-5/6}\, dx$



(b) $\displaystyle\int_{0}^{\frac{\pi}{2}} \frac{\cos x}{\sqrt[5]{\sin x}}\, dx$



Ex 3: Evaluate these integrals.

| | |
|---|---|
| (a) $\int_0^5 \frac{x}{9-x^2}\,dx$   | (b) $\int_0^{27} \frac{x^{1/3}}{x^{2/3}-9}\,dx$ |