Math1210 Midterm 2 Extra Review Key

1. Let $y=2x^3-sec(\pi x)+\sqrt{x}$. If x changes from 1 to 1.035, approximately how much does y change?

Answer: 0.2275

2. Find the indicated derivative of the given functions.

(a)
$$D_x(\tan(4x^2+5x-1)\cos^2(3x))$$

$$= (sec^2(4x^2+5x-1))(8x+5)\cos^2(3x) + \tan(4x^2+5x-1)2\cos(3x)(-\sin(3x))(3)$$

$$\frac{d}{dx} \left(\frac{x^4-3x^2+1}{x^3-\sqrt[4]{x}}\right)^5$$

(b)
$$= 5 \left(\frac{x^4 - 3x^2 + 1}{x^3 - \sqrt[4]{x}} \right)^4 \left(\frac{(x^3 - \sqrt[4]{x})(4x^3 - 6x) - (x^4 - 3x^2 + 1)(3x^2 - \frac{1}{4\sqrt[4]{x^3}})}{(x^3 - \sqrt[4]{x})^2} \right)^4$$

(c)
$$f'(1)$$
 if $f(x)=(2x-\frac{1}{x})^3(4x^3-2)^4$ Answer: 528

(d)
$$\frac{dy}{dx}$$
 given $2x^4y + y^3 = 2x^2 - 6x$ Answer: $\frac{dy}{dx} = \frac{4x - 6 - 8x^3y}{2x^4 + 3y^2}$
(e) $f^{('')}(x)$ for $f(x) = (3x - 4)^{\frac{2}{5}}$ Answer: $f^{('')}(x) = \frac{1296}{125}(3x - 4)^{-13/5}$

(e)
$$f'''(x)$$
 for $f(x)=(3x-4)^{\frac{2}{5}}$ Answer: $f'''(x)=\frac{1296}{125}(3x-4)^{-13/5}$

- 3. A softball diamond has the shape of a square with sides 40 ft. long. If a player is running from second base to third base at a speed of 20 ft/sec, at what rate is her distance from home plate changing when she is 30 ft from third base? Answer: -12 ft/sec
- 4. For (i) $f(x) = \frac{x}{1+x^2}$ and separately for (ii) $f(x) = (x^2-3)^2$, answer the following questions.

 (a) Fill in the sign line for f'(x).



(b) Find all local min and max **point(s)**.

max point: $(1, \frac{1}{2})$ min point: $(-1, -\frac{1}{2})$

(d) Find all x-values of inflection point(s).

inflection point: (0,0),
$$\left(-\sqrt{3}, \frac{-\sqrt{3}}{4}\right)$$
 , $\left(\sqrt{3}, \frac{\sqrt{3}}{4}\right)$

(e) Identify all critical **points** on the closed interval [-2, 2].

on closed interval, critical points are: (-2, -2/5), $(1, \frac{1}{2})$, $(-1, -\frac{1}{2})$, and (2, 2/5)

- 5. Show that the tangent lines to the curves $v^2 = 4x^3$ and $2x^2 + 3y^2 = 14$ at (1, 2) are perpendicular to each other. (ask in class)
- 6. A 13 foot ladder is leaning against a vertical wall. If the bottom of the ladder is moving away from the wall at a constant rate of 0.5 ft/sec, how fast is the top of the ladder sliding down the wall when the top of the ladder is 5

feet above the ground? Answer: -1.2 ft/sec

7. Find
$$f^{(4)}(x)$$
 for $f(x) = (3x-7)^{\frac{5}{3}}$ Answer: $f^{(4)}(x) = 40(3x-7)^{\frac{-7}{3}}$

8. Let $y = \frac{2}{x}$. If x changes from 1 to 1.05, approximately how much does y change?

Answer: -0.1

9. The area of an equilateral triangle is decreasing at a rate of 2 square centimeters per second. Find the rate at which the length of a side is changing when the area of the triangle is $100\sqrt{3}$ square

centimeters. (Note: Area of equilateral triangle with side length x is $A = \frac{\sqrt{3}}{4}x^2$.)

Answer: $\frac{-2}{10\sqrt{3}}$ cm/sec