

### Math1210 Midterm 2 Extra Review Key

1. Let  $y = 2x^3 - \sec(\pi x) + \sqrt{x}$ . If  $x$  changes from 1 to 1.035, approximately how much does  $y$  change?

Answer: 0.2275

2. Find the indicated derivative of the given functions.

(a) 
$$D_x(\tan(4x^2 + 5x - 1)\cos^2(3x))$$

$$= (\sec^2(4x^2 + 5x - 1))(8x + 5)\cos^2(3x) + \tan(4x^2 + 5x - 1)2\cos(3x)(-\sin(3x))(3)$$

$$\frac{d}{dx}\left(\frac{x^4 - 3x^2 + 1}{x^3 - \sqrt[4]{x}}\right)^5$$

(b) 
$$= 5\left(\frac{x^4 - 3x^2 + 1}{x^3 - \sqrt[4]{x}}\right)^4 \left(\frac{(x^3 - \sqrt[4]{x})(4x^3 - 6x) - (x^4 - 3x^2 + 1)(3x^2 - \frac{1}{4\sqrt[4]{x^3}})}{(x^3 - \sqrt[4]{x})^2}\right)$$

(c)  $f'(1)$  if  $f(x) = (2x - \frac{1}{x})^3(4x^3 - 2)^4$       Answer: 528

(d)  $\frac{dy}{dx}$  given  $2x^4y + y^3 = 2x^2 - 6x$       Answer:  $\frac{dy}{dx} = \frac{4x - 6 - 8x^3y}{2x^4 + 3y^2}$

(e)  $f'''(x)$  for  $f(x) = (3x - 4)^{\frac{2}{5}}$       Answer:  $f'''(x) = \frac{1296}{125}(3x - 4)^{-13/5}$

3. A softball diamond has the shape of a square with sides 40 ft. long. If a player is running from second base to third base at a speed of 20 ft/sec, at what rate is her distance from home plate changing when she is 30 ft from third base?      Answer: -12 ft/sec

4. For (i)  $f(x) = \frac{x}{1+x^2}$  and separately for (ii)  $f(x) = (x^2 - 3)^2$ , answer the following questions.

(a) Fill in the sign line for  $f'(x)$ .       $\begin{array}{ccccccc} & & - & & + & & - \\ & & \dots\dots\dots & | & \dots\dots\dots & | & \dots\dots\dots \\ & & -1 & & 1 & & \end{array}$

(b) Find all local min and max **point(s)**.  
 max point:  $(1, \frac{1}{2})$     min point:  $(-1, -\frac{1}{2})$

(c) Fill in the sign line for  $f''(x)$ .       $\begin{array}{ccccccc} & & - & & + & & - & & + \\ & & \dots\dots\dots & | & \dots\dots\dots & | & \dots\dots\dots & | & \dots\dots\dots \\ & & -\sqrt{3} & & 0 & & \sqrt{3} & & \end{array}$

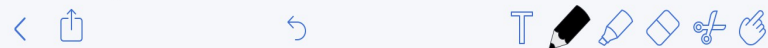
(d) Find all x-values of inflection point(s).  
 inflection point:  $(0,0)$ ,  $(-\sqrt{3}, \frac{-\sqrt{3}}{4})$ ,  $(\sqrt{3}, \frac{\sqrt{3}}{4})$

(e) Identify all critical **points** on the closed interval  $[-2, 2]$ .  
 on closed interval, critical points are:  $(-2, -2/5)$ ,  $(1, \frac{1}{2})$ ,  $(-1, -\frac{1}{2})$ , and  $(2, 2/5)$

5. Show that the tangent lines to the curves  $y^2=4x^3$  and  $2x^2+3y^2=14$  at  $(1, 2)$  are perpendicular to each other.

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S) answer:

① tangent line to  $y^2=4x^3$  at  $(1,2)$

$$2y \frac{dy}{dx} = 12x^2 \quad \text{at } (1,2)$$

$$\frac{dy}{dx} = \frac{6x^2}{y} \Rightarrow m = \frac{6(1^2)}{2} = 3$$

② tangent line to  $2x^2+3y^2=14$  at  $(1,2)$

$$4x + 6y \frac{dy}{dx} = 0$$

$$6y \frac{dy}{dx} = -4x$$

$$\frac{dy}{dx} = \frac{-2x}{3y} \Rightarrow m = \frac{-2(1)}{3(2)} = -\frac{1}{3}$$

Since slope of 3 is negative reciprocal of slope of  $-\frac{1}{3}$ , then the two tangent lines are perpendicular.

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36



6. A 13 foot ladder is leaning against a vertical wall. If the bottom of the ladder is moving away from the wall at a constant rate of 0.5 ft/sec, how fast is the top of the ladder sliding down the wall when the top of the ladder is 5 feet above the ground? Answer: -1.2 ft/sec

7. Find  $f^{(4)}(x)$  for  $f(x)=(3x-7)^{\frac{5}{3}}$  Answer:  $f^{(4)}(x)=40(3x-7)^{\frac{-7}{3}}$

8. Let  $y=\frac{2}{x}$ . If  $x$  changes from 1 to 1.05, approximately how much does  $y$  change?

Answer: -0.1

9. The area of an equilateral triangle is decreasing at a rate of 2 square centimeters per second. Find the rate at which the length of a side is changing when the area of the triangle is  $100\sqrt{3}$  square centimeters. (Note: Area of equilateral triangle with side length  $x$  is  $A=\frac{\sqrt{3}}{4}x^2$ .)

Answer:  $\frac{-2}{10\sqrt{3}}$  cm/sec