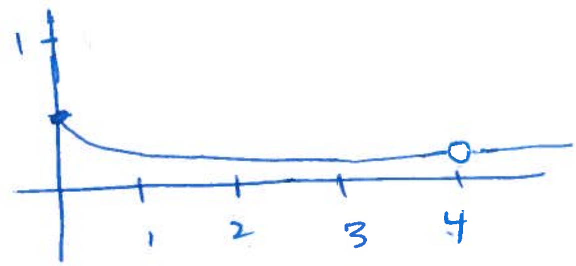
1.5 Limits

Ex Explore what happens to this function

$y = f(x) = \dfrac{\sqrt{x} - 2}{x - 4}$ as x gets close to 4.

x	y
3	$\frac{\sqrt{3}-2}{-1} \approx 0.2679$
3.5	$\frac{\sqrt{3.5}-2}{-0.5} \approx 0.2583$
3.9	$\frac{\sqrt{3.9}-2}{-0.1} \approx 0.251582$
3.999	$\frac{\sqrt{3.999}-2}{3.999-4} \approx 0.250016$
⋮	
4	?
⋮	
4.001	~ 0.24998
4.1	~ 0.24846
4.5	~ 0.24264
5	~ 0.23607



- we write this as

$$\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4}$$

- read as "the limit of $\frac{\sqrt{x}-2}{x-4}$ as x goes to 4"

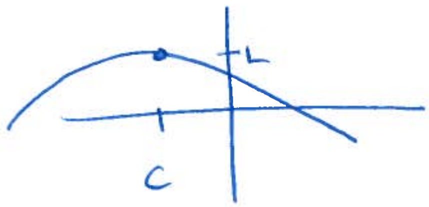
Can we do something algebraically to find this output value?

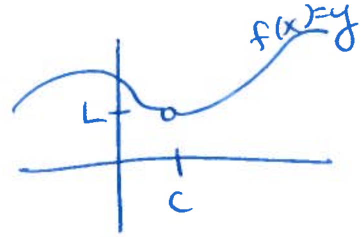
$$\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4}$$

1.5 (cont)

Limit of a fn

$\lim\limits_{x\to c} f(x) = L$ means as x gets close to c, the fn value gets close to L.





Properties of Limits

If $\lim\limits_{x\to c} f(x)$ and $\lim\limits_{x\to c} g(x)$ exist, then

① $\lim\limits_{x\to c} [f(x) \pm g(x)] = \lim\limits_{x\to c} f(x) \pm \lim\limits_{x\to c} g(x)$

② $\lim\limits_{x\to c} (k f(x)) = k \lim\limits_{x\to c} f(x)$, $k \in \mathbb{R}$

③ $\lim\limits_{x\to c} (f(x) g(x)) = \left(\lim\limits_{x\to c} f(x)\right)\left(\lim\limits_{x\to c} g(x)\right)$

④ $\lim\limits_{x\to c} \left(\dfrac{f(x)}{g(x)}\right) = \dfrac{\lim\limits_{x\to c} f(x)}{\lim\limits_{x\to c} g(x)}$ (if $g(x) \neq 0$)

⑤ $\lim\limits_{x\to c} (f(x))^p = \left(\lim\limits_{x\to c} f(x)\right)^p$ (if $\left(\lim\limits_{x\to c} f(x)\right)^p$ exists)

⑥ $\lim\limits_{x\to c} k = k$

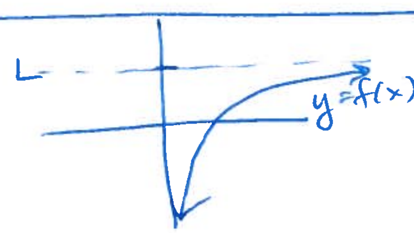
⑦ $\lim\limits_{x\to c} x = c$

Limits at ∞ (these turn into horizontal asymptotes)

$\lim\limits_{x\to\pm\infty} \dfrac{A}{x^k} = 0$ if A and k are constants

$\lim\limits_{x\to\infty} f(x) = L$, $\lim\limits_{x\to-\infty} f(x) = M$

means as x gets huge, the fn value approaches L (as $x\to\infty$) or M (as $x\to-\infty$)



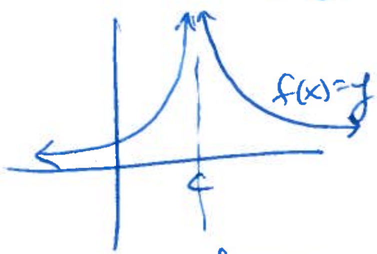
1.5 (cont)

Infinite limits

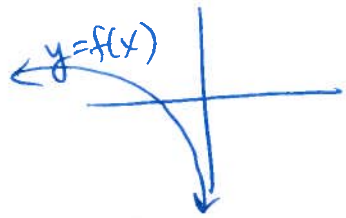
(these end up being vertical asymptotes if c is finite)

$$\lim_{x \to c} f(x) = \pm\infty$$

means as x gets close to c, the fn value increases (or decreases) unendingly



$$\lim_{x \to c} f(x) = \infty$$



$$\lim_{x \to 0} f(x) = -\infty$$

Ex 1 Find limits.

(a) $\displaystyle \lim_{x \to 1} \frac{2x+3}{x+1}$

(b) $\displaystyle \lim_{x \to 9} \frac{\sqrt{x}-3}{x-9}$

1.5 (cont)

Ex2 Find limits.

(a) $\lim_{x \to 2} \frac{x^2+x-6}{x-2}$

(b) ① $\lim_{x \to \infty} (1+x^2)^3$ ② $\lim_{x \to -\infty} (1+x^2)^3$

(c) ① $\lim_{x \to \infty} \frac{1-2x^3}{x+1}$ ② $\lim_{x \to -\infty} \frac{1-2x^3}{x+1}$

1.5 (cont)

Ex 3 Find limits, given this information.

$\lim_{x\to c} f(x) = 2$, $\lim_{x\to c} g(x) = -3$, $\lim_{x\to\infty} f(x) = -4$, $\lim_{x\to\infty} g(x) = 5$

(a) $\lim_{x\to\infty}\left(\sqrt{g(x)} + 3f(x)\right)$

(b) $\lim_{x\to c}\left(\frac{2f(x) - g(x)}{(g(x))^2 + 5f(x)}\right)$

Ex 4 A business manager determines that t months after production begins on a new product, the # of units produced will be P thousand, where

$P(t) = \frac{6t^2+5t}{(t+1)^2}$. What happens to production in the long run?