$D_x(\cos x) = -\sin x$
$D_x(\tan x) = \sec^2 x$
$D_x(\cot x) = -\csc^2 x$
$D_x(\sec x) = \sec x \tan x$
$D_x(\csc x) = -\csc x \cot x$
$D_x(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$ $x \in (-1,1)$
$D_x(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$ $x \in (-1,1)$
$D_x(\tan^{-1} x) = \frac{1}{1+x^2}$
$D_x(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}, |x|>1$
$D_x(\sinh x) = \cosh x$
$D_x(\cosh x) = \sinh x$
$D_x(\operatorname{csch} x) = -\operatorname{csch} x \coth x$
$D_x(\tanh x) = \operatorname{sech}^2 x$
$D_x(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$
$D_x(\coth x) = -\operatorname{csch}^2 x$
$D_x(\sinh^{-1} x) = \frac{1}{\sqrt{x^2+1}}$
$D_x(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$ $x>1$
$D_x(\tanh^{-1} x) = \frac{1}{1-x^2}$ $-1<x<1$
$D_x(\operatorname{sech}^{-1} x) = \frac{-1}{x\sqrt{1-x^2}}$ $0<x<1$

Integrals → +C

$\int \frac{1}{x}dx = \ln|x| + c$
$\int e^x dx = e^x + c$
$\int a^x dx = \frac{1}{\ln a} a^x + c$
$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$
$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}(x) + c$
$\int \frac{1}{1+x^2} dx = \tan^{-1}(x) + c$
$\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1}(x) + c$
$\int \sinh(x) dx = \cosh x + c$
$\int \cosh(x) dx = \sinh x + c$
$\int \tanh(x) dx = \ln|\cosh x| + c$
$\int \tanh(x)\operatorname{sech}(x) dx = -\operatorname{sech}(x) + c$
$\int \operatorname{sech}^2(x) dx = \tanh(x) + c$
$\int \operatorname{csch}^2(x) dx = -\coth(x) + c$
$\int \operatorname{csch}(x)\coth(x) dx = -\operatorname{csch}(x) + c$
$\int \tan(x) dx = -\ln|\cos x| + c$
$\int \cot(x) dx = \ln|\sin x| + c$
$\int \cos(x) dx = \sin(x) + c$
$\int \sin(x) dx = -\cos(x) + c$
$\int \frac{1}{\sqrt{a^2-u^2}} = \sin^{-1}\frac{u}{a} + c$
$\int \frac{1}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + c$

Functions and Identities

$\sin(\cos^{-1} x) = \sqrt{1-x^2}$
$\cos(\sin^{-1} x) = \sqrt{1-x^2}$
$\sec(\tan^{-1} x) = \sqrt{1+x^2}$
$\tan(\sec^{-1} x) = \sqrt{x^2-1}$ if $x \geq 1$
$= -\sqrt{x^2-1}$ if $x \leq -1$
$\sinh^{-1} x = \ln(x + \sqrt{x^2+1})$
$\cosh^{-1} x = \ln(x + \sqrt{x^2-1})$ $x \geq 1$
$\tanh^{-1} x = \frac{1}{2}\ln\left[\frac{1+x}{1-x}\right]$ $-1<x<1$
$\operatorname{sech}^{-1} x = \ln\left[\frac{1+\sqrt{1-x^2}}{x}\right]$ $0<x\leq 1$
$\sinh x = \frac{e^x - e^{-x}}{2}$ $\cosh x = \frac{e^x + e^{-x}}{2}$

TRIG IDENTITIES

$\sin^2 x + \cos^2 x = 1$
$1 + \tan^2 x = \sec^2 x$
$1 + \cot^2 x = \csc^2 x$
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$
$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$
$\sin 2x = 2\sin x \cos x$
$\cos 2x = \cos^2 x - \sin^2 x$
$\cosh^2 x - \sinh^2 x = 1$

$\sin^2 x = \frac{1 - \cos 2x}{2}$
$\cos^2 x = \frac{1 + \cos 2x}{2}$

Product Formulas

$\sin(mx)\cos(nx) = \frac{1}{2}[\sin(m+n) + \sin(m-n)]$
$\sin(mx)\sin(nx) = -\frac{1}{2}[\cos(m+n) - \cos(m-n)]$
$\cos(mx)\cos(nx) = \frac{1}{2}[\cos(m+n) + \cos(m-n)]$

Strategies For Integration

1. U-Substitution
2. Integration by Parts
3. Trig Substitution
4. Partial Fraction Decomposition
5. Trig Integrals needing pythagorean identities or half angle formulas

INDETERMINATE & Determinate cases

$\frac{0}{0}$ $\frac{\infty}{\infty}$ $0 \cdot \infty$ $\infty - \infty$ 0^0 ∞^0 1^∞ INDET
$1^\infty \to 1$ $0^\infty \to 0$ $\infty^\infty \to \infty$ $\infty \cdot \infty \to \infty$
$\infty + \infty \to \infty$ $\frac{c}{\infty} \to 0$ $\frac{\infty}{c} \to \infty$ DETER.

Series

1. N^{th} test (Divergence)
2. Geometric Sums
3. P-Series (Power Series)
4. LCT - Limit Comparison test (Power over Power)
5. R.T. - Ratio Test (Factorials & Exponentials)
6. OCT. Ordinary Comparison test (a_n & b_n)
7. Partial sums (collapsing Sums)
8. Integral test (if you can integrate it)

Power Series

Geometric Series

$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ $x \in (-1,1)$
$\sum_{n=0}^{\infty} \frac{1}{n+1} x^{n+1} = -\ln(1-x)$
$\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} x^{n+1} = \ln(1+x)$
e^x
$\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$

Arctan x
$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} = \arctan x$ $\forall x \in (-1,1)$

Sinh and Cosh
$\sum_{n=1}^{\infty} \frac{x^{2n-1}}{(2n-1)!} = \sinh(x)$
$\sum_{n=1}^{\infty} \frac{x^{2n}}{(2n)!} = \cosh(x)$

convergence set

Force it less than 1
if 0 then its all real #s

Taylor & Maclaurin Series

Uniqueness therem
$f(x) = C_0 + C_1(x-a) + C_2(x-a)^2 + C_3(x-a)^3 + \ldots$
$C_n = \frac{f^{(n)}(a)}{n!}$

Taylors Formula w/ Remainder
$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} + \ldots$
$\frac{f^{(n)}(a)}{n!}(x-a)^n + R_n(x)$
$R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}$
(where c is some value between x & a)

Taylors Series (about a)
Maclaurin Series a=0.

Binomial Series

$\binom{p}{k} = {}_pC_k = \frac{p!}{k!(p-k)!}$

Triangle Inequality

$|a+b| \leq |a| + |b|$

Polar Coordinates

$r = \sqrt{x^2+y^2}$ $x = r\cos\theta$
$\tan\theta = \frac{y}{x}$ $y = r\sin\theta$
rect → Polar Polar → Rect.

Polar Equations

1. lines $r = \frac{d}{\cos(\theta_0 - \theta)}$
2. Circles (generic Circle Equation)
$a^2 = r^2 + c^2 - 2rc\cos(\theta - \theta_0)$
if c=a then it passes through origin
$r = 2a\cos(\theta - \theta_0)$
3. Conics (Parabolas, Hyperbolas, Ellipses)
$r = \frac{ed}{1 + e\cos(\theta - \theta_0)}$ e>1 Hyp, e=1 Para, e<1 elipse

Graphs of Polar Equations

Limacon $r = a \pm b\cos\theta$ $r = a \pm b\sin\theta$ ($a \geq b$, $a=b$, $a<b$)
Lamniscates $r^2 = \pm a\cos(2\theta)$ $r^2 = \pm a\sin(2\theta)$
Rose $r = a\cos(n\theta)$ $r = a\sin(n\theta)$ n-odd leaves 2n-even
Spiral $r = a\theta$

5. $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \ldots$
6. $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \ldots$
7. $\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \frac{x^9}{9!} + \ldots$
8. $\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \frac{x^8}{8!} + \ldots$
9. $(1+x)^p = 1 + \binom{p}{1}x + \binom{p}{2}x^2 + \binom{p}{3}x^3 + \binom{p}{4}x^4 \ldots$

Factorials

1! = 1
2! = 2
3! = 6
4! = 24
5! = 120
6! = 720
7! = 5040
8! = 40320
9! = 362880

Miscellaneous

$\lim_{h\to 0}(1+h)^{\frac{1}{h}} = e$
$\lim_{n\to\infty}(1 - \frac{1}{n})^n = \frac{1}{e}$

Integration by Parts
$\int u\,dv = uv - \int v\,du$

First Order of linear Differential Equations

$\frac{dy}{dx} + P(x)y = Q(x)$
(multiply by integrating factor $e^{\int P(x)dx}$)

Kirchoff's law

$L\frac{dI}{dt} + RI = E(t)$ R = resistance, L = inductance, E = voltage, I = current

Euler's Method

$x_n = x_{n-1} + h$
$y_n = y_{n-1} + hf(x_{n-1}, y_{n-1})$

Half Life
$y = y_0 e^{rt}$

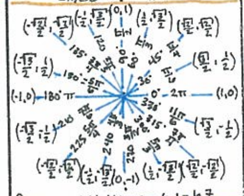
Calculus in Polar Coordinates

$A = \frac{1}{2}\int_\alpha^\beta r^2 d\theta$ or $A = \frac{1}{2}\int_\alpha^\beta [f(\theta)]^2 d\theta$

Tangent line Slope

$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$

Unit Circle



Quizes and Midterms – 6.1 – 6.7

1. Find $\frac{dy}{dx}$ $y = \ln(x^4 + \sqrt{x^2+4})^3 = y = 3\ln(x^4 + \sqrt{x^2+4})$
$\frac{dy}{dx} = \frac{3(4x^3 + \frac{2x}{2\sqrt{x^2+4}})}{x^4 + \sqrt{x^2+4}}$

2. $\int_0^1 \frac{5t^3+3t^2}{5t^4+4t^3-1}dt$ $v = 5t^4+4t^3-1$ $t=0$ $v=-1$; $du = (20t^3+12t^2)dt$ $t=1$ $v=8$; $\frac{1}{4}du = 5t^3+3t^2dt$
$\frac{1}{4}\int_{-1}^{8} \frac{1}{u}du = \frac{1}{4}\ln|u|\Big]_{-1}^{8} = \frac{1}{4}(\ln 8 - \ln 1) = \frac{1}{4}\ln 8$

$f(x) = -2x^3 - \tan x - 5x + 3$ $(f^{-1})(3)$
$f'(x) = -6x^2 - \sec^2 x - 5$ monotonically decreasing
$3 = -2x^3 - \tan x - 5x + 3 \Rightarrow x = 0$
$(f^{-1})'(3) = \frac{1}{f'(0)} = \frac{1}{-6(0) - \sec^2 0 - 5} = \frac{1}{0-1-5} = -\frac{1}{6}$

4. $f^{-1}(x)$ for $f(x) = \frac{x^3-5}{3x^3+2}$ $= y(3x^3+2) = x^3 - 5$; $x^3(3y-1) = -2y-5$; $x^3 = \frac{-2y-5}{3y-1}$ $\sqrt[3]{\frac{2x+5}{1-3x}}$
Switch from y to x at the end

$y = \pi^{\frac{3}{x}} + (\frac{3}{x})^\pi$ $\frac{dy}{dx} = \pi^{\frac{3}{x}}(\ln\pi)(-\frac{3}{x^2}) + 3^\pi(-\pi)x^{-\pi-1}$
exponential Power

$y = (x^5 - x)^{\ln x} = \ln y = \ln x \ln(x^5 - x)$
$D_x(\ln y) = D_x(\ln x \ln(x^5 - x))$
$\frac{1}{y}y' = \frac{1}{x}\ln(x^5-x) + \ln x\left(\frac{5x^4-1}{x^5-x}\right)$
$y' = (x^5-x)^{\ln x}\left(\frac{\ln(x^5-x)}{x} + \frac{\ln x(5x^4-1)}{x^5-x}\right)$

$\int_{-1}^0 4^{2x+1}dx = \frac{4^{2x+1}}{2\ln 4}\Big|_{-1}^0 = \frac{1}{2\ln 4}(4^1 - 4^{-1})$
$= \frac{1}{2\ln 4}(4 - \frac{1}{4}) = \frac{1}{2\ln 4}(\frac{15}{4})$
$= \frac{15}{8\ln 4}$
a^x

$\int \frac{e^{2x}}{e^{2x}+1}$ $u = e^{2x}+1$, $du = e^{2x}(2)dx$, $\frac{1}{2}du = e^{2x}dx$; $\frac{1}{2}\int \frac{1}{u}du = \frac{1}{2}\ln|u| + c$
$= \frac{1}{2}\ln(e^{2x}+1) + c$
$= \ln\sqrt{e^{2x}+1} + c$

Half Life – of 3 years, 8grams to start, How much after 5years. $y = 8e^{rt}$
$y = y_0 e^{rt}$
$4 = 8e^{3r}$
$\frac{1}{2} = e^{r3} = \ln\frac{1}{2} = 3r$
$r = \frac{1}{3}\ln .5$
$y = 8^{\frac{1}{3}\ln .5t} = 2.5198$ grms

Euler's Method $h = .2$ $y' = -4xy$ [-1,0] through Point (-1,2)
$h = .2$ $y_n = y_{n-1} + h(-4x_{n-1}y_{n-1}) = y_{n-1}(x_n, y_n)$
$= y_{n-1}(1 - 0.8x_{n-1})$

-1	2
-.8	3.6
-.6	5.904

$\frac{1}{x^3}\frac{dy}{dx} = 3 + \frac{y}{x^4}$ $y=3$ $x=1$ $P = -\frac{1}{x}$ $Q = 3x^3$
$\frac{dy}{dx} = 3x^3 + \frac{y}{x} = \frac{dy}{dx} - \frac{1}{x}y = 3x^3$
$\frac{1}{x}\frac{dx}{dx} - \frac{1}{x^2}y = 3x^2 = \int(\frac{1}{x}y) = \int 3x^2 dx$
$y = x^4 + Cx = y = x^4 + 2x$

$y = (x^{\ldots})$ $\frac{1}{y}y' = (-\sin x)(\ln(x^2+\frac{\pi}{2})) + \cos x\left(\frac{2x - \frac{\pi}{2}}{x^2 + \ldots}\right)$
$y' = (x^2 \ldots)^{\cos x}\left[\ldots\right]$

$y = (\sqrt{2x} + 4x)^{\ldots} e^{\ldots}$
$y = (\ln x)^3 - \frac{x^2}{\ln x^2}$ $= 3(\ln x)^2(\frac{1}{x}) - \frac{2\ln x(2x) - x^2(\frac{2}{x})}{9(\ln x)^2}$
$y' = \frac{3}{x}(\ln x)^2 - \frac{9x^2(\ln x - 3x^2)}{9(\ln x^3)^2}$

$f(x) = 2x^3 - 9 + \tan^{-1}x + x$ $(f^{-1}(9))$
$f'(x) = 6x^2 + \frac{1}{1+x^2} + 1 > 0$ $\forall x \in \mathbb{R}$
$(f^{-1})'(9) = \frac{1}{f'(0)} = \frac{1}{0+1+1} = \frac{1}{2}$

$\int \frac{2x^3 - 1}{12x^4 - 16x + 5}$ $u = 12x^4 - 16x + 5$; $du = (48x^3 - 16)dx$; $\frac{1}{16}du = (3x^3 - 1)dx$
$\frac{1}{16}\int \frac{1}{u}du = \frac{1}{16}\ln|u| + c = \frac{1}{16}\ln|12x^4 - 16x + 5| + c$

$\int \frac{2\sec^2 x}{1+\tan^2 x}dx$ $u = \tan x$, $du = \sec^2 x dx$; $2\int \frac{1}{1+u^2}du$; $2\tan^{-1}u + c$; $2\tan^{-1}(\tan x) + c$

$\int_3^5 5^{2x-3}dx = \frac{1}{2}\left(\frac{5^{2x-3}}{\ln 5}\right)\Big|_3^5 = \frac{1}{2\ln 5}(5^7 - 5^3)$
$= \frac{1}{2\ln 5}(125 - \frac{1}{5}) = \frac{1}{2\ln 5}\left(\frac{625-1}{5}\right) = \frac{312}{5\ln 5}$

$x\frac{dy}{dx} + 2y = \sqrt{x}$ $\int (x^2 y)' = \int x^{1/2}dx$
$\frac{dy}{dx} + \frac{2}{x}y = \frac{1}{\sqrt{x}}$ $x^2 y = \frac{2}{3}x^{3/2} + c$
$x^2\frac{dy}{dx} + 2xy = x^{3/2}$ $y = \frac{2}{3}x^{-\frac{1}{2}} + \frac{c}{x^2}$
$-\frac{1}{3} = \frac{2}{3} + c$ $c = -1$
$y = \frac{2}{3}\sqrt{x} - \frac{1}{x^2}$

$f^{-1}(x)$ $f(x) = 5x^2 - 10x + 7$ complete Square
$f(x) = 5(x^2 - 2x + 1) + 7 - 5 = 5(x-1)^2 + 2$
$f^{-1}(x) = \sqrt{\frac{x-2}{5}} + 1$

Population 12,000 in 1980 to 18,000 in 1992 → 2020?
let T=0 in 1980 $P = P_0 e^{rt}$ $P = 12000e^{rt}$
$18000 = 12000e^{12r} = \frac{3}{2} = e^{12r}$
$\ln(1.5) = 12r \Rightarrow r = \frac{1}{12}\ln(1.5)$ $P = 12000e^{\frac{t}{12}\ln(1.5)}$
$P = 12000e^{\frac{40}{12}\ln(1.5)(40)}$
$= 46360.93 \approx 46361$

Eulers Method $h = .2$ $y' = 2x^2y - x + 1$ [0,1], $y(0) = 2$
$y_{n-1} + h(2x_{n-1}^2 y_{n-1} - x_{n-1} + 1)$
0 / .2 / .4 / .6: 2 / 2.2 / 2.3952 / 2.6249

Quizes and Midterms 6.8 – 8.4

$\sin(2\cos^{-1}(\frac{12}{13}))$ let $\theta = \cos^{-1}(\frac{12}{13}) \Rightarrow \cos\theta = \frac{12}{13}$
$\sin(2\cos^{-1}(\frac{12}{13})) = \sin(2\theta) = 2\sin\theta\cos\theta$
$= 2(\frac{5}{13})(\frac{12}{13}) = \frac{120}{169}$

$y = \tan^{-1}(\tanh x) + 4\sinh^{-1}(7x)$
$\frac{1}{1+\tanh^2 x}(\operatorname{sech}^2 x) + 8\sinh(7x)\cosh(7x)(7)$

$y = \cosh x$ $x = 0$ and $x = \ln 2$
$A = \int_0^{\ln 2} \cosh x\,dx = \sinh x\Big|_0^{\ln 2}$
$= \sinh(\ln 2) - \sinh(0) = \frac{e^{\ln 2} - e^{-\ln 2}}{2} - \frac{e^0 - e^0}{2}$
$= \frac{2 - \frac{1}{2}}{2} - 0 = 1 - \frac{1}{4} = \frac{3}{4}$

$\int \frac{x^3}{x^8+1}dx$ $u = x^4$, $du = 4x^3dx$; $\frac{1}{4}\int \frac{1}{u^2+1}du$
$\frac{1}{4}\arctan u + c = \frac{1}{4}\arctan(x^4) + c$

$\int \frac{x^3}{x^4+1}dx$ $u = x^4+1$, $du = 4x^3dx$; $\frac{1}{4}\int \frac{1}{u}du$
$\frac{1}{4}\ln|u| + c$
$\frac{1}{4}\ln|x^4+1| + c$

$\int \ln(5x-1)dx$ $u = \ln(5x-1)$, $du = \frac{5}{5x-1}dx$; $dv = dx$, $v = x$
$x\ln(5x-1) - \int \frac{5x}{5x-1}dx = x\ln(5x-1) - \int 1 + \frac{1}{5x-1}dx$
long division
$x\ln(5x-1) - x - \frac{1}{5}\ln|5x-1| + c$

$\int \cos(3x)\sin(4x)dx$
$\cos(3x)\sin(4x) = \frac{1}{2}(\sin(7x) + \sin(x))$
$\frac{1}{2}\int \sin(7x) + \sin(x)dx = \frac{1}{2}\left(-\frac{\cos(7x)}{7} + -\cos x\right) + c$
$= -\frac{1}{14}\cos(7x) - \frac{1}{2}\cos x + c$

$\int \frac{2x^2+7x-8}{x+4}$ $x+4\overline{)2x^2+7x-8}$ ... $2x-1 - \frac{4}{x+4}$
$\int 2x - 1 - \frac{4}{x+4}dx$
$x^2 - x - 4\ln|x+4| + c$

$\int x^2 e^x dx$ $u = x^2$, $du = 2x\,dx$; $dv = e^x dx$, $v = e^x$
$x^2e^x - 2\int xe^x dx$ $u = x$, $du = dx$; $dv = e^x dx$, $v = e^x$
$x^2e^x - 2xe^x + 2e^x + C$

$\int \frac{x+3}{\sqrt{x-5}} + \ldots = 3\ln|x| + \ln|x-3| \ldots$
$\int \frac{3x+2}{\sqrt{x-5}}$ $u = \sqrt{x-5} \Rightarrow u^2 = x - 5$; $du = \frac{1}{2\sqrt{x-5}}$, $u^2 + 5$, $2u\,du = dx$
$\int (3(u^2+5)+2)(2u)du$
$2\int 3u^3 + 17u\,du = 2(\frac{3}{4}u^4 + \frac{17}{2}u^2) + c$
$\frac{3}{2}(x-5)^2 + 17(x) + c$

$\int \frac{\sqrt{x^2-16}}{x}dx$ $x = 4\sec\theta$, $dx = 4\sec\theta\tan\theta d\theta$
$\sqrt{x^2-16} = \sqrt{16\sec^2\theta - 16} = \sqrt{16(\sec^2\theta - 1)} = 4\tan\theta$
$\int \frac{\sqrt{x^2-16}}{3x}dx = \frac{1}{3}\int \frac{4\tan\theta}{4\sec\theta}(4\sec\theta\tan\theta)d\theta$
$\frac{4}{3}\int \tan^2\theta d\theta = \frac{4}{3}\int \sec^2\theta - 1 d\theta = \frac{4}{3}(\tan\theta - \theta) + c$
$\frac{4}{3}\left[\frac{\sqrt{x^2-16}}{4} - \sec^{-1}\left(\frac{x}{4}\right)\right] + c$

$\int \frac{5e^x}{25e^{2x}+4}dx$ $u = 5e^x$, $du = 5e^x dx$; $\int \frac{du}{u^2+4} = \frac{1}{2}\tan^{-1}(\frac{u}{2}) + c$
$= \frac{1}{2}\arctan\left(\frac{5e^x}{2}\right) + c$

$\lim_{x\to\frac{\pi}{2}} \left(\frac{\ln(\sin x)}{3(x - \frac{\pi}{2})^2}\right) \overset{L}{=} \lim_{x\to\frac{\pi}{2}} \left(\frac{\frac{\cos x}{\sin x}}{6(x-\frac{\pi}{2})}\right) = \left(\frac{\cot x}{6(x-\frac{\pi}{2})}\right)$
$\overset{L}{=} \lim \left(\frac{-\csc^2 x}{6}\right) = \frac{-1}{6\sin^2(\frac{\pi}{2})} = -\frac{1}{6}$

$\lim_{x\to 1} \frac{x^2+2x-3}{2x^2+3x-5} \overset{L}{=} \lim_{x\to 1} \frac{2x+2}{4x+3} = 4/7$

$\lim_{x\to 0} (\sin x)^x = 0$ 0^0 case → Determinate

$\lim_{x\to\infty} (\ln(x+2) - \ln(x-3)) = \lim_{x\to\infty} \left(\ln\left(\frac{x+2}{x-3}\right)\right)$
$\ln\left(\lim \left(\frac{x+2}{x-3}\right)\right) \overset{L}{=} \ln\left(\lim \left(\frac{1}{1}\right)\right) = \ln(1) = 0$

$\int_1^\infty \frac{2}{x(\ln x)^2}dx$ $u = \ln x$, $du = \frac{1}{x}dx$; $x = e$ $u = 1$; $x = \infty$ $u = \infty$
$\int_1^\infty \frac{2}{u^2}du = \lim_{b\to\infty} 2\int_1^b u^{-2}du \ldots = \lim (2(\frac{u^{-1}}{-1}))\Big|_1^b$
$= \lim_{b\to\infty} \left(\frac{1}{(1)}\right) = (\frac{1}{b}) - (-1) = 1$

$\int_3^6 \frac{1}{(x-5)^{4/3}}dx = \int_3^5 (x-5)^{-4/3}dx + \int_5^6 (x-5)^{-4/3}dx$
$\lim_{c\to 5^-} \int_3^c (x-5)^{-4/3}dx + \lim_{b\to 5^+} \int_b^6 (x-5)^{-4/3}dx$
$\lim_{c\to 5^-} \frac{-3}{\sqrt[3]{x-5}}\Big|_3^c + \lim_{b\to 5^+} \frac{-3}{\sqrt[3]{x-5}}\Big|_b^6$
$\lim \frac{-3}{\sqrt[3]{c-5}} + \frac{3}{\sqrt[3]{-2}} + \frac{-3}{\sqrt[3]{1}} - \lim \frac{-3}{\sqrt[3]{d-5}}$
$-\infty$ Diverges

$\int_{-1}^\infty e^{-2x}dx = \lim_{c\to\infty} \frac{e^{-2x}}{-2}\Big|_{-1}^c$
$-\frac{1}{2}\left(\lim_{c\to\infty} \frac{1}{e^{2c}} - \frac{1}{e^{2}}\right) = \frac{1}{2e^2}$

$\int_0^1 \frac{1}{\sqrt{1-x^2}} = \lim_{b\to 1^-} \int_0^b \frac{1}{\sqrt{1-x^2}} = \lim_{b\to 1^-} \sin^{-1}(x)\Big|_0^b$
$\lim_{b\to 1^-} \sin^{-1}(b) - \sin^{-1}(0) = \sin^{-1}(1) = \frac{\pi}{2}$

$\int \frac{\sqrt{x-2}}{x+3}dx$ $u = \sqrt{x-2}$ ⇒ $u^2 = x - 2$; $du = \frac{1}{2\sqrt{x-2}}$
$2\sqrt{x-2}\,du = dx$; $2u\,du = dx$; $\int \frac{u(2u)}{u^2+5}du$
$U^2 + 5\overline{)2u^2}$; $\int 2 - \frac{10}{u^2+5}du$
$2u - 10\int \frac{1}{u^2+5}du$
$= 2u - \frac{10}{\sqrt{5}}\tan^{-1}\left(\frac{u}{\sqrt{5}}\right)$
$2\sqrt{x-2} - \frac{10}{\sqrt{5}}\tan^{-1}\left(\frac{\sqrt{x-2}}{\sqrt{5}}\right) + c$

$\int \frac{5x^2+3x+7}{(x-1)^2(x+4)}dx = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+4}$
$5x^2+3x+7 = A(x-1)(x+4) + B(x+4) + C(x-1)^2$
$x = 1$: $15 = 5B \Rightarrow B = 3$
$x = -4$: $75 = 25C \Rightarrow C = 3$
$x = 0$: $7 = -4A + 12 + 3 \Rightarrow -8 = 4A = 2$
$\int \frac{2}{x-1} + \frac{3}{(x-1)^2} + \frac{3}{(x+4)}dx$
$2\ln|x-1| - \frac{3}{x-1} + 3\ln|x+4| + C$

$\lim_{x\to 0} (\cos x)^{\frac{1}{x^2}} = \lim e^{\ln(\cos x)^{1/x^2}}$
$e^{\lim \frac{\ln(\sin(\cos x))}{x^2}} \overset{L}{=} \lim \frac{5(-\sin x)}{\frac{\cos x}{6x^5}}$
$\overset{L}{=} \lim_{x\to 0} \frac{-\sec^2 x}{30x^4} \to \frac{-5}{0} \to -\infty$

$\int_0^{\frac{\pi}{4}} \frac{\sec^2(x)}{(3-\tan x)^2}dx$ $u = 3 - \tan x$, $du = -\sec^2 x$; $\frac{\pi}{4} = \frac{9-\sqrt{3}}{3}$; $\frac{\pi}{4} = 2$
$\int_{\frac{9-\sqrt{3}}{3}}^{2} -\frac{1}{u^2}du = -\int u^{-2}du = -\frac{u^{-1}}{-1}$
$\frac{1}{2} - \frac{1}{9-\frac{\sqrt{3}}{3}} = \frac{1}{2} - \frac{3}{9-\sqrt{3}}$

$\int_{-\infty}^{-1} \frac{4}{(4x-3)^2}dx = \lim_{b\to-\infty} \int_b^{-1} \frac{4}{(4x-3)^2}dx$
$\lim_{b\to-\infty} \frac{4(4x-3)^{-1}}{-4}\Big|_b^{-1} = -\frac{4}{4}\left(\lim_{b\to-\infty} \frac{1}{4(-1)-3} - \frac{1}{4b-3}\right)$
$= -\frac{4}{4}(-\frac{1}{7} - 0) = \frac{4}{28}$

$\int 2x^2(25-x^2)^{\frac{3}{2}}dx$ $x = 5\sin\theta$, $dx = 5\cos\theta$
$\int \frac{25\sin^2\theta}{(25\cos^2\theta)^{3/2}}5\cos\theta d\theta$ $25 - x^2 = 25 - 25\sin^2\theta$ $= 25(1 - \sin^2\theta)$ $= 25\cos\theta$
$\int \tan^2\theta d\theta = 2\int \sec^2\theta - 1 d\theta$
$= 2\tan\theta - 2\theta + C$
$\frac{x}{\sqrt{25-x^2}}$ $\frac{x}{5} = \sin\theta$
$\frac{2x}{\sqrt{25-x^2}} - 2\arcsin\left(\frac{x}{5}\right) + C$

$\int_3^6 (x-4)^{-\frac{5}{3}}dx = \int_3^4 (x-4)^{-\frac{5}{3}}dx + \int_4^6 (x-4)^{-\frac{5}{3}}dv$
$\lim_{c\to 4^-} \int_3^c (x-4)^{-\frac{5}{3}}dx + \lim_{d\to 4^+} \int_d^6 (x-4)^{-\frac{5}{3}}dx$
$\lim_{c\to 4^-} \left(-\frac{3}{2}(x-4)^{-\frac{2}{3}}\right) + \lim_{d\to 4^+} \left(-\frac{3}{2}(x-4)^{-\frac{2}{3}}\right)\Big|_d^6$
$\lim_{c\to 4^-} \frac{-3}{2(c-4)^{2/3}} \ldots$
$-\infty$ diverges

$\lim_{x\to 0} \left(\frac{\tan x - \sin x}{4x^3}\right) \overset{L}{=} \lim_{x\to 0} \left(\frac{\sec^2 x - \cos x}{12x^2}\right)$
$\overset{L}{=} \lim_{x\to 0} \frac{2\sec^2 x\tan x + \sin x}{24x}$
$\overset{L}{=} \lim_{x\to 0} \frac{4\sec^2 x\tan^2 x + 2\sec^4 x + \cos x}{24}$
$= \frac{0+2+1}{24} = \frac{3}{24} = \frac{1}{8}$

$\int y^2\sin(y)dy$ $u = y^2$, $du = 2y\,dy$; $dv = \sin y\,dy$, $v = -\cos y$
$= -y^2\cos y + 2\int y\cos y\,dy$ $u = y$, $du = dy$; $dv = \cos y\,dy$, $v = \sin y$
$= -y^2\cos y + 2(y\sin y - \int \sin y\,dy)$
$= -y^2\cos y + 2y\sin y - 2\cos y + c$

9.1 – 9.7

$\sum_{n=1}^\infty \frac{\sqrt[3]{4n^3-2n}}{3n-2}$ $n = 2$ $\sqrt[3]{28}/4$; $n = 3$ $\sqrt[3]{102}/7$
$\lim_{n\to\infty} \frac{\sqrt[3]{4n^3-2n}}{3n-2} \sim \frac{\sqrt[3]{4n^3}}{3n} = \frac{\sqrt[3]{4}n}{3n} = \frac{\sqrt[3]{4}}{3}$
Converges

$\frac{2}{3}, \frac{8}{9}, \frac{18}{27}, \frac{32}{81}, \ldots$ $a_n = 2(-1)^n\left(\frac{n^2}{3^n}\right)$
$\lim_{n\to\infty} |a_n| = \lim \frac{2n^2}{3^n} \overset{L}{=} 0$ Converges

Ball dropped 150ft rebounds 3/5
$150(1 + \frac{3}{5} + \frac{9}{25} + \ldots)$
$150 + 150(2)\left(\sum \left(\frac{3}{5}\right)^n\right)$ geometric Series
$150 + 300\sum \left(\frac{3}{5}\right)\left(\frac{3}{5}\right)^{n-1}$ $r = \frac{3}{5} < 1$ converges
$= 150 + 300\left(\frac{\frac{3}{5}}{1-\frac{3}{5}}\right) = 150 + 300\left(\frac{3}{2}\right) = 600$ ft

$\sum_{n=1}^\infty \frac{\sqrt[3]{4n^3-2n}}{3n-2}$ Nth test Diverges
$\lim_{n\to\infty} \frac{\sqrt[3]{4n^3-2n}}{3n-2} = \frac{\sqrt[3]{4}}{3}$

$\left(\frac{5}{1} - \frac{5}{2}\right) + \left(\frac{10}{2} - \frac{10}{3}\right) + \left(\frac{15}{3} - \frac{15}{4}\right) + \left(\frac{20}{4} - \frac{20}{5}\right) \ldots$
$\sum_{n=1}^\infty \frac{6n}{n^2 - \frac{1}{2}}$ LCT $b_n = \frac{5}{n}$
$\lim_{n\to\infty} \frac{5n}{n^2 - \frac{1}{2}}\left(\frac{n}{5}\right) = \lim_{n\to\infty} \frac{n^2}{n^2 - \frac{1}{2}} = 1$
Diverges by LCT
$\sum_{i=1}^\infty \frac{2}{3^i} + \frac{2}{3^i} = \sum_{i=1}^\infty 3\left(\frac{1}{2^i}\right) + \sum_{i=1}^\infty 2\left(\frac{1}{5}\right)^i$
P-Series P>1 converges; geometric $r = \frac{1}{5} < 1$ converges

$\sum_{n=2}^\infty \frac{2n}{(2n-1)!} = \lim_{n\to\infty} \frac{2^{n+1}}{(2(n+1)-1)!} \cdot \frac{(2n-1)!}{2^n}$
$= \lim_{n\to\infty} \frac{2(2n-1)!}{(2n+1)!} = \lim \frac{2(2n-1)!}{(2n+1)(2n)(2n-1)!}$
$= \lim_{n\to\infty} \frac{2}{2n(2n+1)} = 0 < 1$ converges
Ratio test

$\sum_{k=1}^\infty 3ke^{-2k^2}$ I Test $u = -2k^2$ $x=1$ $u=2$
$\int_1^\infty \frac{-3}{8}e^u du$ ...
$-\frac{3}{8}\int e^u = -\frac{3}{8}\left[\lim_{b\to\infty} e^b - e^2\right] = \frac{3}{8}e^2$
finite Convergence

AST $\lim_{n\to\infty} \frac{5^n}{\sqrt{n^4(n^2+1)}} \to \infty$ Diverges

$\sum_{n=1}^\infty \frac{\cos(n\pi)}{n^4} = \sum_{n=1}^\infty \frac{(-1)^n}{n^4}$ $\sum|a_n|$
Power Series $= \sum_{n=1}^\infty \frac{1}{n^4}$
P=4>1
Absolute Convergence

$\frac{x-2}{1\cdot 2} + \frac{(x-2)^2}{2\cdot 3} + \frac{(x-2)^3}{3\cdot 4} + \frac{(x-2)^4}{4\cdot 5} + \ldots$
$= \sum_{n=1}^\infty \frac{(x-2)^n}{n(n+1)}$ ART $\lim_{n\to\infty} \frac{|x-2|^{n+1}}{(n+1)((n+1)+1)} \cdot \frac{n(n+1)}{|x-2|^n}$
$= \lim_{n\to\infty} \frac{|x-2|n}{n+2} = |x-2|\lim_{n\to\infty} \frac{n}{n+2} \sim 1$
$|x-2| < 1 = 1 < x < 3$ Power Series Converges
check end Pts
$x = 3$ $\sum_{n=1}^\infty \frac{1}{n(n+1)}$ $b_n = \frac{1}{n^2}$ LCT $= \lim \frac{1}{n(n+1)} \cdot \frac{n^2}{1}$
$= \lim \frac{n^2}{n^2+n} = 1$
$x = 1$ $\sum_{n=1}^\infty \frac{(-1)^n}{n(n+1)}$ by AST converges
[1, 3]

$f(x) = \frac{5x^4}{(1+3x)^2} = 5x^4\left(\frac{1}{(1+3x)^2}\right)$
$= \sum_{n=1}^\infty 5x^4(n)(3x)^{n-1} = \sum_{n=1}^\infty 5n(3^{n-1})(x^{n+3})$
$|3x| < 1$ $|x| < 1/3 = R$

$f(x) = \sin x + \sin^2 x + \sin^3 x + \sin^4 x + \ldots$
$\sum_{n=1}^\infty (\sin x)^n - 1 = \frac{1}{1 - \sin x} - 1$
$= \frac{\sin x}{1 - \sin x}$ $|\sin x| < 1$ $x \neq \frac{(2m+1)\pi}{2}$ $\forall m \in \mathbb{Z}$

$\int \frac{e^x\sin(e^x)}{\cos(e^x)}dx$ $v = \cos e^x$; $-dv = \sin(e^x)e^x dx$
$= -\int \frac{du}{u} = -\ln|u| + c = -\ln|\cos(e^x)| + c$

$\sum_{k=3}^\infty \frac{3k-2}{9+\sqrt{k^3}}$ (all Positive) LCT $b_n = \frac{k}{\sqrt{k^3}} = \frac{1}{k^{1/2}}$ diverges $P = \frac{1}{2} < 1$
$\lim_{n\to\infty} \left(\frac{3k-2}{9+\sqrt{k^3}}\right) \cdot \frac{\sqrt{k}}{1} \sim \lim_{n\to\infty} \frac{3k^{3/2}}{k^{3/2}} = 3$
Diverges

$\sum_{n=1}^\infty \sqrt[n]{2n}$ Nth term test
$\lim_{n\to\infty} (2n)^{\frac{1}{n}} = e^{\lim \frac{\ln(2n)}{n}}$
$\overset{L}{=} \lim_{n\to\infty} \frac{\ln(2n)}{n} \overset{L}{=} \lim_{n\to\infty} \frac{\frac{1}{n}}{1} = 0$
$\Rightarrow e^{\lim \frac{\ln(2n)}{n}} = e^0 = 1$ Diverges

$\sum_{n=1}^\infty \frac{(-1)^n 4n^2}{2n^3 - n}$ (decreasing)
AST $\lim_{n\to\infty} \frac{4n^2}{2n^3 - n} = 0$ conditional convergence
LCT $b_n = \frac{2}{n}$ $\lim_{n\to\infty} \frac{4n^2}{2n^3 - n} \cdot \frac{n}{2} = \frac{4n^3}{4n^3} = 1$
Diverges; conditionally Converges

$\sum_{n=1}^\infty \frac{n^4 5^n}{(2n+1)!}$ Positive Series
RT $\lim \frac{(n+1)^4 5^{(n+1)}}{(2(n+1)+1)!} \cdot \frac{(2n+1)!}{n^4 5^n}$
$= \frac{(n+1)^4(5)(2n+1)!}{n^4(2n+3)(2n+2)(2n+1)!}$
$= \lim \frac{5(n+1)^4}{n^4(2n+3)(2n+2)} \sim \frac{5n^4}{4n^6} \to 0 < 1$ converges

$\sum_{n=1}^\infty \frac{3n^2}{e^{n^3}}$ Positive decreasing continuous IT
$\int_1^\infty \frac{3x^2}{e^{x^3}}dx = \int_1^\infty 3x^2 e^{-x^3}dx$ $u = -x^3$, $du = -3x^2$; $x = 1$ $u = -1$; $x = \infty$ $u \to -\infty$
$-\int_{-1}^{-\infty} e^u du = -\lim_{b\to-\infty} e^u\Big|_{-1}^b$
$= -\left[\lim \frac{1}{e^{b}} - e^{-1}\right] = \frac{1}{e}$ Converges

$0.0525252 \ldots$
$\frac{52}{1000}\left(\frac{1}{10}\right) = \sum_{n=1}^\infty \left(\frac{52}{1000}\right)\left(\frac{1}{10}\right)^{n-1}$
$\frac{\frac{52}{1000}}{1 - \frac{1}{100}} = \frac{52}{1000 - 100} = \frac{52}{990} = \frac{26}{495}$

$\ln\left(\frac{3}{5}\right) + \ln\left(\frac{5}{7}\right) + \ln\left(\frac{7}{9}\right) + \ldots$ $\sum_{n=1}^\infty \ln\left(\frac{2n-1}{2n+1}\right)$
$(\ln 3 - \ln 5) + (\ln 5 - \ln 7) + (\ln 7 - \ln 9) \ldots$
$(\ln 3 - \ln(3n+2)) \to \infty$
Diverges Collapsing Sum

$\overset{L}{=} |x-1|\lim_{n\to\infty} \frac{2(2^n)\ln 2}{4(2^n)\ln 2} = \frac{|x-1|}{2} < 1$
$|x-1| < 2$ $R = 2$

$a_n = \frac{\sqrt[3]{n^2 - n^3 + 6}}{3n+1}$ $n=0 = \sqrt[3]{6}$; $n=1$ $\sqrt[3]{6}/4$; $n=2$ $\sqrt[3]{6}/7$
$\lim_{n\to\infty} \frac{\sqrt[3]{n^2 - n^3 + 6}}{3n+1} = \frac{\sqrt[3]{n^3}}{3n} = -\frac{1n}{3n}, -\frac{1}{3}$ Converges

$\sum_{n=0}^\infty \frac{(-2)^{n+1}(x-1)^n}{2n+3}$ ART
$\lim_{n\to\infty} \frac{2^{n+2}|x-1|^{n+1}}{2(n+1)+3} \cdot \frac{2n+3}{2^{n+1}|x-1|^n}$
$|x-1|\lim_{n\to\infty} \frac{2(2n+3)}{2n+5} = 2|x-1|\lim_{n\to\infty} \frac{2n+3}{2n+5}$
$2|x-1| < 1 \Rightarrow |x-1| < 1/2$
$1/2 < x < 3/2$
Test end Pts $x = \frac{1}{2}$ $\sum_{n=0}^\infty \frac{(-2)^{n+1}(-\frac{1}{2})^n}{2n+3} = \frac{(-2)(-2)^n(-\frac{1}{2})^n}{2n+3}$
$= -\sum_{n=0}^\infty \frac{2}{2n+3}$ LCT $b_n = \frac{1}{n}$ diverges
$\lim_{n\to\infty} \frac{2}{2n+3} \cdot \frac{n}{1} = 1$
$x = \frac{3}{2}$ $\sum_{n=0}^\infty \frac{-2(-2)^n(\frac{1}{2})^n}{2n+3} = \sum_{n=0}^\infty \frac{-2(-1)^n}{2n+3}$
$= -2\sum_{n=0}^\infty \frac{(-1)^n}{2n+3}$ AST $\lim \frac{1}{2n+3} \to 0$ converges
$\left(\frac{1}{2}, \frac{3}{2}\right]$

$f(x) = \frac{3x^3}{1 - \frac{1}{2}x}$ $g(\wp) = \frac{1}{1-\wp}$ $\sum_{n=1}^\infty \wp^n$
$\frac{1}{1 - \frac{1}{2}x} = \sum_{n=0}^\infty \left(\frac{1}{2}x\right)^n = \sum_{n=0}^\infty \frac{x^n}{2^n}$
$\Rightarrow \frac{3x^3}{1 - \frac{1}{2}x} = 3x^3\sum_{n=0}^\infty \frac{x^n}{2^n} = \sum_{n=0}^\infty \frac{3x^{n+3}}{2^n}$
$\forall |\frac{1}{2}x| < 1 \Rightarrow |x| < 2$
$R = 2$

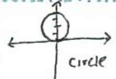
$f(x) = e^{2x} + \sin x - 2x^2$ x^4 terms
$e^{2x} = \sum_{n=0}^\infty \frac{(2x)^n}{n!} = 1 + 2x + \frac{4x^2}{2!} + \frac{8x^3}{3!} + \frac{16x^4}{4!}$
$\sin x = \sum \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots$
$1 + 2x + \frac{4x^2}{2} + \frac{4}{3}x^3 + \frac{2}{3}x^4 + x - \frac{1}{6}x^3 - 2x^2$
$1 + 3x + \frac{7}{6}x^3 + \frac{2}{3}x^4 + \ldots$

$f(x) = \sqrt{2x+3}$ 3 term $f(0) = \sqrt{3}$
$f'(x) = \frac{1}{2}(2x+3)^{-1/2}(2)$ $f'(0) = \frac{1}{\sqrt{3}}$
$f''(x) = -\frac{1}{2}(2x+3)^{-3/2}(2)$ $f''(0) = \frac{-1}{3\sqrt{3}}$
$f'''(x) = -3/2(2x+3)^{-5/2}(2)$ $f'''(0) = \frac{-1}{3\sqrt{3}}$
$\sqrt{3} + \frac{1}{\sqrt{3}}x - \frac{1}{6\sqrt{3}}x^2 - \frac{1}{18\sqrt{3}}x^3 \ldots$

$(x-1)$ $f(x) = e^x$ through $(x-1)^3$
$f(x) = \sum_{n=0}^\infty \frac{f^{(n)}(1)}{n!} = \sum_{n=0}^\infty \frac{e}{n!}(x-1)^n$
$e + e(x-1) + \frac{e}{2}(x-1)^2 + \frac{e}{6}(x-1)^3 + \ldots$

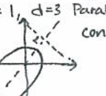
$R_6(x)$ $a = 1$ for $f(x) = e^{-2x}$ $|R_6(0.5)|$
$f'(x) = -2e^{-2x}$ $R_6(x) = \frac{(-2)^7 e^{-2c}}{7!}(x-1)^7$
$f''(x) = 4e^{-2x}$ $|R_6(0.5)| = \left|\frac{2^7(-\frac{1}{2})^7}{e^{2c}7!}\right|$
$f'''(x) = (-2)^3 e^{-2x}$
$f^{(4)}(x) = (-2)^4 e^{-2x}$ $= \frac{1}{e^{2c}7!} \leq \frac{1}{e(7!)}$
$R_6(x) = \frac{(-2)^7 e^{-2c}}{7!}(x-1)^7$
$|R_6(0.5)| \leq \frac{1}{7!e}$

$r = 4\sin\theta = 0$
$r = 4\sin\theta$
$r^2 = 4r\sin\theta$
$x^2 + y^2 = 4y$
$x^2 + (y^2 - 4y + 4) = 0 + 4$
$x^2 + (y-2)^2 = 4$



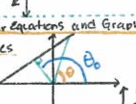
$(-3, \sqrt{3})$ $r^2 = a + 3 = 12$
$r = \pm\sqrt{12} = \pm 2\sqrt{3}$
$\tan\theta = \frac{\sqrt{3}}{-3} = \frac{1}{-\sqrt{3}} = \frac{1/2}{-\sqrt{3}/2}$ $\theta = \frac{5\pi}{6}, -\frac{\pi}{6}$
$(2\sqrt{3}, \frac{5\pi}{6})$ or $(-2\sqrt{3}, -\pi/6)$

$r = \frac{\ldots}{3 + 3\cos(\theta - \frac{\pi}{4})}$
$r = \frac{9}{3(1 + \cos(\theta - \frac{\pi}{4}))} = \frac{3}{1 + \cos(\theta - \frac{\pi}{4})}$
$e = 1$, $d = 3$ Parabola

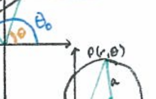


Polar Equations and Graphs

Lines



Circles

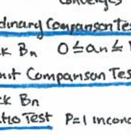


Conics

Series Conditions

Divergence – Nth test if 0 might converge
Integral test → # or converges; ±∞ DNE Diverges
Positive, Continuous, non increasing
P-Series → P>1 converges; P≤1 diverges
Ordinary Comparison Test
Pick B_n $0 \leq a_n \leq b_n$
Limit Comparison Test
Pick B_n
Ratio Test P=1 inconclusive
P<1 converges P>1 $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \infty$

Find Area $r = 3 + 3\sin\theta$



θ	r
0	3
π	3
π/2	6
3π/2	0

$A = 2\left(\frac{1}{2}\right)\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} r^2 d\theta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (3 - 3\sin\theta)^2 d\theta$
$= \int_{-\frac{\pi}{2}}^{\pi/2} 9 + 18\sin\theta + 9\sin^2\theta d\theta$
$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 9 + 18\sin\theta + \frac{9}{2}(1 - \cos(2\theta))d\theta$
$\frac{27}{2}\theta - 18\cos\theta - \frac{9}{4}\sin(2\theta)\Big|_{-\pi/2}^{\pi/2}$
$\frac{27}{2}\left(\frac{\pi}{2} - -\frac{\pi}{2}\right) - 18\left(\cos\left(\frac{\pi}{2}\right) - \cos\frac{\pi}{2}\right) - \frac{9}{4}(\sin(\pi) - \sin(\pi))$
$\frac{27}{2}\pi$

Tangent line Slope

$r = 2 - 3\sin\theta$
$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = 0 = \frac{2\cos\theta - 6\cos\theta\sin\theta}{\ldots}$
$0 = 2\cos\theta - 6\cos\theta\sin\theta$
$0 = 2\cos\theta(1 - 3\sin\theta)$
$\cos\theta = 0$ $-3\sin\theta = -1$
$\pi/2, -\pi/2$ $\sin\theta = 1/3$
$\theta = \pm\pi/2 + 2n\pi$ $n \in \mathbb{Z}$
$\theta = \arcsin\left(\frac{1}{3}\right) + 2n\pi$ $n \in \mathbb{Z}$