4.4 Properties of logarithms

★ see table pg 230 in book

$x \in \mathbb{R}$, $a > 0$, $a \neq 1$, $m > 0$, $n > 0$

1. $\log_a a^x = x$
2. $\log_a a = 1$
3. $\log_a 1 = 0$
4. $a^{\log_a x} = x \quad (x > 0)$
5. $\log_a (mn) = \log_a m + \log_a n$
6. $\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$
7. $\log_a m^n = n \log_a m$

Proof (5)

let $u = \log_a m$ and $v = \log_a n$

then $a^u = m$ and $a^v = n$ by defn.

$\Rightarrow \log_a (mn) = \log_a (a^u a^v)$

$= \log_a (a^{u+v})$

$= u + v$

$= \log_a m + \log_a n$ //

(7) Let $u = n \log_a m$

then $\frac{u}{n} = \log_a m$

$\Leftrightarrow \quad a^{u/n} = m$

$\Rightarrow \log_a (m^n) = \log_a (a^{u/n})^n$

$= \log_a a^u$

$= u$

$= n \log_a m$. //

4.4 (cont)

Ex1 Use log properties to expand.

(a) $\ln\left(\frac{x^2}{x+1}\right)$

(b) $\log_3\left(x^3\sqrt{x-2}\right)$

(c) $\log\left(\frac{y^4}{(y-2)^6}\right)$

Ex2 Use log properties to condense.

(a) $\log_4 8 - \frac{1}{2}\log_4 5 + \log_4 3$

(b) $2(\ln x - \ln(x+5))$

(c) $\log(2x+1) - \frac{1}{3}\log(x-1)$

4.4 (cont)

Ex 3 Evaluate (w/o calculator)

(a) $\log_7(49) + \log_5(125) - \log_2(64)$

(b) $\log_4\left(\frac{1}{64}\right) + \ln(e^7) - \log_5 1$

Ex 4 If $\log_b x = 1.2$, $\log_b y = 3.1$ $\log_b z = 11.1$,

evaluate $\log_b\left(\frac{x}{y}\right) - \log_b(z^2 x)$

4.5 Logarithmic and Exponential Equations

Strategies to Solve Eqns:

Logarithmic	Exponential
① get logs all on one side of eqn	① Isolate exponential
2) condense using log properties	② use defn of log to rewrite as log eqn
3) use defn of log to rewrite in exponential form	③ Continue solving.
4) continue solving.	(* you don't have to check answers because there is no restriction on domain.)
5) check all answers. (This must happen because of restricted domain.)	

Ex 1 Solve. $4^{x+2} = 64$

4.5 (cont)

Ex 2 Solve.

(a) $2e^{x}+3=13$

(b) $5^{x+6}-4=12$

c) $\ln(2x-3)=\ln 11$

(d) $2\log_4 x=5$

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4.5 (cont)

Ex 3 Solve.

(a) $\log_3(2x) - \log_3(x-3) = 1$

(b) $3^{2x} + 3^{x} = 20$

(c) $\log(x^2) = (\log x)^2$

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4.6 Logarithmic & Exponential Business Applications

__Ex1__ If \$1000 is invested at 10% compounded continuously, the future value S at any time t (in years) is given by $S = 1000e^{0.1t}$.

(a) What is the account worth after one year?

(b) How long will it take for the investment to double?

4.6 (cont)

Ex 2 The population of Mathville grows according to the formula $P = P_0 e^{0.03t}$. If the population was 250,000 in the year 2000, estimate the year in which the population reaches 350,000.

4.6 (cont)

Ex 3 Radioactive Iodine-131 has a half-life of 8 days. How long does it take to reduce an initial amount of Iodine-131 to 1% of the initial amount?

Ex 4 The tsunami of 2004 killed over 200,000 people and was measured at $M=9.1$ on the Richter scale. What was its intensity?
(Use $M = \log\left(\frac{I}{I_0}\right)$ where $I_0 = 10^{-3}$ is the zero-level earthquake, or the minimum intensity that can be felt.)