3.1 Quadratic Eqns in One Variable

Defn Quadratic Eqn can be written in form

$ax^2+bx+c=0 \qquad a,b,c \in \mathbb{R}$.

Ex 1 Solve $5x^2-32=x^2+8$

Ex 2 Solve $2x(5x+6)=16$

Strategies to Solve

① Square Root Technique

② Factor Technique

③ Completing the Square

④ Quadratic Formula

3.1 (cont)

Ex 3 Solve $y^2+y-4=0$

Ex 4 $x^2+4=6x$

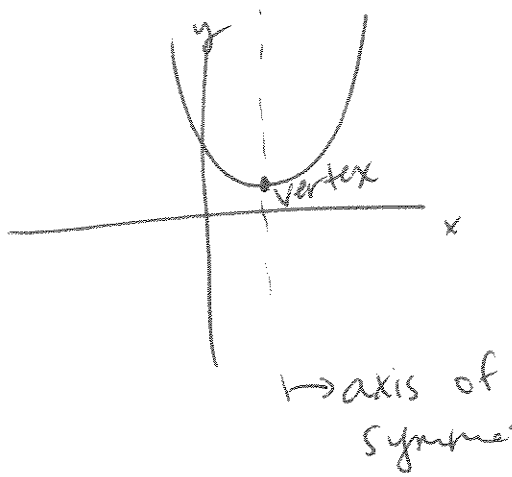
3.1 (cont)

Ex 5

$$\frac{1}{x-10} - \frac{1}{x-9} = 1$$

3.2 Parabolas: Quadratic Equations in Two Variables

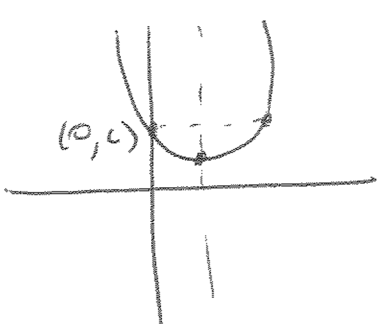
Quadratic Fn $\Rightarrow$ $y = f(x) = ax^2 + bx + c$
(a quadratic eqn in two variables)
when we graph all the solutions to this, the points form a parabola.



For $y = ax^2 + bx + c$,
if $a > 0$, $\cup$ concave up
if $a < 0$, $\cap$ concave down

Let's figure out where the vertex is (algebraically) so we can always find it.

if we plug in $x = 0$, we get pt on y-axis $\Rightarrow$
$x = 0 \Rightarrow y = a(0^2) + b(0) + c \Leftrightarrow y = c$, i.e. parabola goes thru $(0, c)$



We can see, by symmetry of parabola, that there is another pt whose y-value is c.

3.2 (cont)

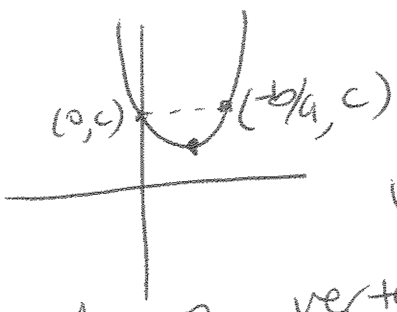
$c = ax^2+bx+c \iff 0 = ax^2+bx$

$0 = x(ax+b)$

$x=0$, or $ax+b=0$

$ax=-b$

$x=-b/a$



you can see (from symmetry) that x-value of vertex is halfway between 0 and $-b/a$, i.e. $x = -b/2a$.

$\Rightarrow$ $\boxed{\text{vertex at } \left(-b/2a,\ f(-b/2a)\right)}$

axis of symmetry $x = -b/2a$

Ex 1 For $y = -2x^2-4x+6$

(a) find vertex

(b) Is the vertex a min or max pt?

3.2 (cont)

Ex 2 For $y = x^2 - 6x + 9$,

(a) find vertex.

(b) Is it a min or max pt?

(c) find zeros of graph.

(d) Find the axis of symmetry.

(e) sketch graph.

3.2 (cont)

Ex 3 If 100 ft of fencing is used to enclose a rectangular yard, find the area function. Find the dimensions of the rectangle that maximize area.

3.3 Quadratic
Business Applications

Supply Demand + Market Equilibrium

Ex 1 If the supply function for a commodity is $p = q^2 + 8q + 20$ and the demand function is $p = 100 - 4q - q^2$, find the equilibrium quantity and equilibrium price. (Sketch both curves.)

3.3 (cont)

Ex 2 For the last example, if an \$8 tax is placed on production & passed through the supplier, find the new equilibrium pt.

3.3 (cont)

Break-Even Pts and Maximization

Ex 3 If a company has total costs $C(x) = 1600 + 1500x$ and total revenue is $R(x) = (1600 - x)x$, find the break even pts.

Break Even pts occur when

$R(x) = C(x)$

$\iff P(x) = 0$

3.3 (cont)

Ex 4 Find maximum revenue given
$R(x) = 1600x - x^2$.

Ex 5 Suppose a company has fixed costs of \$300 and variable costs of $\frac{3}{4}x + 1460$ dollars per unit, where x = total # units produced. Suppose further that its selling price is $1500 - \frac{1}{4}x$ dollars per unit.
(a) Find break even pts.

3.3 (cont)

Ex 5 (cont)

(b) Find max revenue

(c) Find max profit, and price that yields it.

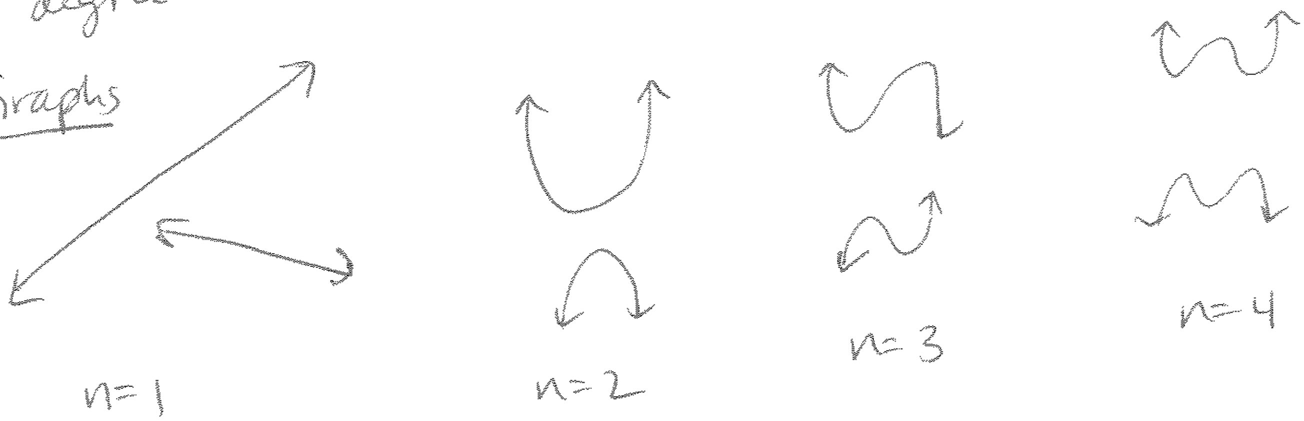
3.4 Polynomial Functions

Polynomial Fn (Defn)

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$$

degree $= n$ $\quad (a_n \neq 0)$

Graphs



Ex 1 For these polynomials, write in standard form. What is its degree and leading coefficient? General graph shape?

(a) $4x - 12 - 2x^3 - x^2$

(b) $3x^7 - 14x + 3x^2 - 4x^4 - 5$

3.4 (cont)

Ex 2 For these polynomials, answer the following.

(a) degree
(b) zeros
(c) y-intercept
(d) x-intercept
(e) sketch graph

① $f(x) = x^4 - 8x^2 + 16$

② $g(x) = 2x^3 - 2x^2 - 4x$

3.4 (cont)

Ex 3 For these piecewise functions, fill in the points + sketch the graph.

(a) $f(x) = \begin{cases} 4 & x \geq 3 \\ |x| & -3 \leq x < 3 \\ -1 & x < -3 \end{cases}$

x	-4	-3	0	1	3	4
y						

(b) $g(x) = \begin{cases} x+5 & x \geq 1 \\ -2x+8 & x < 1 \end{cases}$

x	y
1	
0	
-1	
2	
3	