3.7 Transformations of Graphs

<table>
<thead>
<tr>
<th>Shift</th>
<th>Reflection</th>
<th>Stretch/Shrink</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h(x) = f(x) \pm c$</td>
<td>$g(x) = -f(x)$</td>
<td>$k(x) = cf(x)$, $c \in \mathbb{R}$</td>
</tr>
<tr>
<td>$h(x) = f(x \pm c)$</td>
<td>$g(x) = f(-x)$</td>
<td>$k(x) = f(cx)$, $c \in \mathbb{R}$</td>
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**Example 1**

Describe the transformation of $f(x) = -(x-2)^2 + 3$ compared to the base graph of $y = x^2$. Sketch the graph of $f(x)$. 
Ex 2. Describe x-form and sketch graph.

(a) \( q(x) = -2|x-3| + 1 \)

(b) \( h(x) = 4(x+2)^3 - 3 \)
3.7 (cont)

Ex 3 Given this graph, sketch the indicated transformed graph.

(a) $f(-x) + 1$

(b) $f(x+1)$

(c) $-f(x) + 1$
3.8 Combination of Functions

4.1 Inverse Functions

Fns
- Addition \((f + g)(x) = f(x) + g(x)\)
- Subtraction \((f - g)(x) = f(x) - g(x)\)
- Multiplication \((fg)(x) = f(x)g(x)\)
- Division \(\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}\) (as long as \(g(x) \neq 0\))
- Composition \((f \circ g)(x) = f(g(x))\) Read "f of g of x"

Ex 1 Given \(f(x) = 2x + 5\) \(g(x) = \frac{1}{x^3}\), find

(a) \((f \circ g)(x)\)
(b) \((f + g)(1)\)
(c) \((g \circ f)(1)\)
(d) \(\left(\frac{f}{g}\right)(x)\)
3.8 § 4.1 (cont.)

Ex 2  Given  \( f(x) = x^2 - 1 \)  \( g(x) = \frac{x}{2} \)  \( h(x) = \sqrt{x - 1} \), find

(a) \((h \circ f)(x)\)

(b) \((g - h)(1)\)

(c) \((h \cdot f)(3)\)

(d) \(g(h(x))\)

(e) \(h(f(g(x)))\)
Inverse Fns

An inverse fn basically "undoes" what the original fn did to the input $x$.

* notation: $f^{-1}(x)$ read "f inverse of x"

$$f^{-1}(f(x)) = f(f^{-1}(x)) = x$$

**Ex 3** Are $f(x) = 5x - 1$ and $g(x) = \frac{x+1}{5}$ inverse fns?

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Does every fn have an inverse? No!

A fn that has an inverse passes the horizontal line test (by its graph).

one-to-one; every input has exactly one output
and every output has exactly one input.
Inverse fn graph is mirror image of original fn across line $y = x$.

Does $y = x^2$ have an inverse fn?
Can we restrict its domain so it does have an inverse?

Strategy to find inverse

1. "Pants" Technique
2. Standard Technique

Ex 5 Find inverse of $f(x) = 4(x-1)^3$
Ex 6. Find universe of \( f(x) = \sqrt[3]{\frac{x+1}{2x+3}} \)

Ex 7. Are \( f(x) = 2\sqrt{x} - 1 \) and \( g(x) = \frac{1}{4}(x+1)^2 \) inverses of each other?