3.5 Polynomial Functions

Polynomial Function (Defn)

\[ f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0 \]

degree = \( n \)  \( (a_n \neq 0) \)

Ex. 1. For these polynomials, write in standard form.
What is its degree and leading coefficient?
General graph shape?

(a) \( 4x - 12 - 2x^3 - x^2 \)

(b) \( 3x^2 - 14x + 3x^2 - 4x^4 - 5 \)
3.5 (cont)

Ex 2. For these polynomials, answer the following:

(a) degree
(b) zeros
(c) y-intercept
(d) x-intercept
(e) sketch graph

1. \(f(x) = x^4 - 8x^2 + 16\)

2. \(g(x) = 2x^3 - 2x^2 - 4x\)
Ex 3 For these piecewise functions, fill in the points and sketch the graph.

(a) \( f(x) = \begin{cases} 
4 & x \geq 3 \\
|x| & -3 \leq x < 3 \\
-1 & x < -3 
\end{cases} \)

(b) \( g(x) = \begin{cases} 
x + 5 & x \geq 1 \\
-2x + 8 & x < 1 
\end{cases} \)
3.6 Rational Functions

Definition: Rational Function \( f(x) = \frac{n(x)}{d(x)} \)
where \( n(x) \) and \( d(x) \) are polynomials.

Asymptotes

1. VA (vertical asymptote)
   - Lines \( x = a \) where \( d(a) = 0 \)
   - Find VA by seeing restricted values in domain
   - Cannot touch or cross VA

2. HA (horizontal asymptote)
   - Line \( y = b \) such that \( b = \lim_{x \to \pm\infty} f(x) \)
   - Graph approaches HA as \( x \) gets huge (in either direction)
   - Graph can touch or cross HA when \( x \) is “small”

To graph rational fn:

1. Find domain
   - Find VA
   - Find HA

2. Find \( y \)-intercept and \( x \)-intercept(s)

3. Plot intercept pts and at least one pt on all sides of VAs.

4. Fill in graph with smooth curves that approach asymptotes.
3.6 (cont)

Ex 1: Analyze and graph.

(a) \( f(x) = \frac{2+x}{1-x} \)

(b) \( f(x) = \frac{10}{x^2+2} \)
3.6 (cont)

(c) \( g(x) = \frac{2x+1}{2x^2-5x-3} \)