2.3 Gauss-Jordan Elimination

Vocab

Augmented matrix: A matrix that represents a system of linear eqns.

Elementary row operators:

1. Switch two rows

2. Multiply any row by a nonzero constant

3. Replace one row with the result of adding it to a nonzero multiple of another row.

Gauss-Jordan Elimination: process for solving a system of linear eqns. using elementary row ops, until we have triangular matrix.

Like this

\[
\begin{bmatrix}
1 & 3 & 4 & | & 5 \\
0 & 1 & 2 & | & 7 \\
0 & 0 & 1 & | & -4 \\
\end{bmatrix}
\]

This is augmented matrix for

\[
\begin{align*}
x + 3y + 4z &= 5 \\
y + 2z &= 7 \\
z &= -4
\end{align*}
\]
2.3 (cont)

Ex 1  Solve.

10x + y = 6
3x + y + 2z = 3
2x - y - 2z = 2

Ex 2

-2x + y = 1
2x - y = 7
Ex 3. Solve. \[3x - 2y + 7z = 0\]
\[x - y + z = 1\]
\[-x + 2y - 3z = -4\]

Ex 4. \[3x - y = 3\]
\[x + z = 3\]
\[2x - y + z = 3\]
Ex 5  Solve

\[ \begin{align*}
    x + y + z &= 1 \\
    x - y - z &= 1 \\
    -x + y - z &= 1
\end{align*} \]
2.4 Inverse Matrices

Def: $A^{-1}$ read "A inverse" (it's not an exponent)

$A^{-1} \cdot A = I = A \cdot A^{-1}$

$A^{-1}$ can only exist for square matrix $A$ (nxn) and is also square nxn.

Example 1: Find $A^{-1}$ for

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Method to find $A^{-1}$ for $A$

① If $A$ is not square, $A^{-1}$ DNE.

② If $A$ is square,
   (a) augment $A$ with identity matrix
   (b) Perform elementary row ops on augmented matrix until the left side is $I$
   (c) What's on the right side is $A^{-1}$.
2.4 (cont)

Ex 2  Find $A^{-1}$.

(a) $A = \begin{bmatrix} 7 & 6 \\ 3/2 & 4 \end{bmatrix}$

(b) $A = \begin{bmatrix} 6 & 0 & 5 \\ -3 & 5 & -3 \\ 7 & 3 & 6 \end{bmatrix}$
Ex3 use \( A^{-1} \) from Ex2(b) to solve

\[
\begin{align*}
6x + 5z &= 1 \\
-3x + 5y - 3z &= 0 \\
7x + 3y + 6z &= 7
\end{align*}
\]

To solve \( Ax = b \)

(\( A \) is \( n \times n \) matrix, \( x \) is \( n \times 1 \) column vector of variables and \( b \) is \( n \times 1 \) column vector of constants), we can left-multiply both sides by \( A^{-1} \),

\[
A^{-1}Ax = A^{-1}b
\]

\[
Ix = A^{-1}b
\]

\[
x = A^{-1}b
\]