5.1 Arithmetic & Geometric Sequences

**Vocab** sequence: \( \{a_n\} \) an ordered list of numbers that form a pattern; it's also a function.

\[ \text{domain} = \mathbb{N} \]

**Arithmetic Sequence**

\[ a_n = a_{n-1} + d , \text{ given } a_1 , \]

(we add same # over + over to get next terms)

\[ n = 2, 3, \ldots , \quad d \neq 0 \]

- \( a_1 \) given
- \( a_2 = a_1 + d \)
- \( a_3 = a_2 + d = a_1 + d + d = a_1 + 2d \)
- \( a_4 = a_3 + d = a_1 + 2d + d = a_1 + 3d \)
- \( a_5 = a_4 + d = a_1 + 3d + d = a_1 + 4d \)
- \( \vdots \)

- \( a_n = a_{n-1} + d = a_1 + (n-1)d \)

**Geometric Sequence**

\[ a_n = d a_{n-1} , \quad n = 2, 3, \ldots , \quad d \neq 0 \]

(we multiply same # over + over to get to next terms in the sequence)

\[ a_1 \) given
- \( a_2 = da_1 , \)
- \( a_3 = da_2 = d(da_1) = d^2a_1 , \)
- \( a_4 = da_3 = d(d^2a_1) = d^3a_1 , \)
- \( a_5 = da_4 = d(d^3a_1) = d^4a_1 , \)
- \( \vdots \)
- \( a_n = d^{n-1} a_1 \)

*These formulas are recursive (they depend on previous terms).

*These formulas are iterative (or direct or explicit) (they don't depend on previous terms).
5.1 (cont)

Ex 1: Classify as arithmetic or geometric and give next 3 terms of sequence.

(a) 10, 7, 4, ...

(b) 2, 6, 18, ...

Ex 2: Find formula for $n^{th}$ term of (a) arithmetic and (b) geometric sequence

(a) $a_1 = 2, d = -3$

(b) $a_1 = -10, d = 2$
5.1 (cont)

Ex 3 Given $a_1 = 2$ and $a_8 = 23$, find the 50th term of this arithmetic sequence.

Ex 4 Given $a_1 = \frac{3}{2}$ and $a_6 = \frac{3}{64}$, find the 20th term of this geometric sequence.
5.1 (cont)

### Arithmetic Sequence Sum

\[ S_n = a_1 + a_2 + \ldots + a_n = ? \]

\[ a_i + a_n = a_1 + (a_i + (n-1)d) \]
\[ = 2a_1 + (n-1)d \]
\[ a_2 + a_{n-1} = 2a_1 + (n-1)d = a_1 + a_n \]
\[ a_3 + a_{n-2} = 2a_1 + (n-1)d = a_1 + a_n \]
\[ \vdots \]

have \( \frac{n}{2} \) groups of this

\[ S_n = \frac{n}{2} (a_1 + a_n) \]

### Geometric Sequence Sum

\[ S_n = a_1 + a_2 + \ldots + a_n = ? \]

\[ S_n = a_1 + da_1 + d^2a_1 + \ldots + d^{n-1}a_1 \]
\[ -dS_n = da_1 + d^2a_1 + d^3a_1 + \ldots + d^n a_1 \]

\[ S_n - dS_n = a_1 - d^n a_1 \]
\[ S_n (1-d) = a_1 (1-d^n) \]

\[ S_n = \frac{a_1 (1-d^n)}{1-d} \]

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**Ex 5.1**

(a) Find sum of first 100 terms of sequence

1, 10, 19, 28, \ldots

(b) Find sum of first ten terms of sequence

3, 6, 12, 24, \ldots
5.2 Simple & Compound Interest

**Simple Interest**
- add same interest to account every period
- if an arithmetic sequence and account balance is the sum

\[ P = \text{principal} = \text{start value} \]

\[ Pr = \text{principal times interest rate = interest that gets added every period} \]

\[ S = P + Pr(t) \]

\[ S = P(1 + rt) \]

\[ P = \text{principal}, \ r = \text{int. rate}, \ t = \# \text{ yrs}, \ S = \text{future acct value} \]

**Compound Interest**
- multiply by same rate every period
- it's a geometric sequence and account balance is the sum

\[ P = \text{principal} = \text{start value} \]

\[ (1 + r) = \text{factor that multiplies by principal every year} \]

\[ S = P(1 + r)^t \]

if we compound \( n \) times per year,

\[ S = P \left(1 + \frac{r}{n}\right)^{nt} \]

\[ n = \# \text{ compounding per year} \]

*see table pg 271-272*

\[ S = Pe^{rt} \]

\[ \text{Continuous Compounding} \]

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5.2 (Cont)

Ex 1  If $10,000 is invested for 4 years at an annual rate of 8.5% (simple), how much will the account be worth at the end of 4 years? Re-compute for compounded interest (compounded once per year).

Ex 2  You borrow $2000 at an interest rate of 20%. How much interest is due in 34 weeks?
Ex 3 If $1000 is invested at 5%:
(a) simple interest
(b) compounded interest
   w/ n = 1
   w/ n = 12,
(c) how long does it take to double?

Ex 4 What is an account worth in 8 years, if we started with $3000 and we get continuous compounding at a rate of 6%?
Ex 5 What amount must be invested now in order to have $1,000,000 for retirement in 45 years if money is compounded quarterly at 9%?

APY (Annual Percentage Yield)

Let $P = \$100$ be invested at 8% interest compounded

(a) quarterly or (b) monthly.

(a) $S = 100 \cdot \left(1 + \frac{0.08}{4}\right)^{4(45)} = 100 \cdot (1.02)^{4} = \$108.24$ after 1 yr

(b) $S = 100 \cdot \left(1 + \frac{0.08}{12}\right)^{12(45)} = \$108,300$ after 1 yr

So investing at 8% compounded quarterly is basically equivalent to investing the same money at 8.24% simple interest.

That's the APY
5.2 (cont)

\[ \text{APY} = \left(1 + \frac{r}{n}\right)^n - 1 \]  \hspace{1cm} (periodic compounding)
\[ \text{APY} = e^r - 1 \]  \hspace{1cm} (continuous compounding)

Ex 6 Which is a better investment deal?
(a) 10\% compounded annually
(b) 9.8\% compounded quarterly
(c) 9.65\% compounded continuously