4.4 Properties of logarithms

\( x \in \mathbb{R}, \ a > 0, \ a \neq 1, \ m > 0, \ n > 0 \)

1. \( \log_a a^x = x \)

2. \( \log_a a = 1 \)

3. \( \log_a 1 = 0 \)

4. \( a^{\log_a x} = x \quad (x > 0) \)

5. \( \log_a (mn) = \log_a m + \log_a n \)

6. \( \log_a \left( \frac{m}{n} \right) = \log_a m - \log_a n \)

7. \( \log_a m^n = n \log_a m \)

**Proof**

Let \( u = \log_a m \) and \( v = \log_a n \)

Then \( a^u = m \) and \( a^v = n \)

by defn.

\[ \Rightarrow \log_a (mn) = \log_a (a^u a^v) = \log_a (a^{u+v}) = u + v = \log_a m + \log_a n \]

Let \( u = n \log_a m \)

then \( \frac{u}{n} = \log_a m \)

\( \Rightarrow \ a^\frac{u}{n} = m \)

\[ \Rightarrow \log_a (m^n) = \log_a (a^{\frac{u}{n}})^n = \log_a a^u = u = n \log_a m \]
4.4 (cont)

Ex 1 Use log properties to expand.

(a) $\ln\left(\frac{x^2}{x+1}\right)$

(b) $\log_3(x^3\sqrt{x-2})$

(c) $\log\left(\frac{y^4}{(y-2)^6}\right)$

Ex 2 Use log properties to condense.

(a) $\log_4 8 - \frac{1}{2} \log_4 5 + \log_4 3$

(b) $2(\ln x - \ln(x+5))$

(c) $\log(2x+1) - \frac{1}{3} \log(x-1)$
4.4 (cont)

Ex.3 Evaluate (w/o calculator)

(a) \( \log_7(49) + \log_5(125) - \log_2(64) \)

(b) \( \log_y\left(\frac{1}{64}\right) + \ln(e^x) - \log_5 1 \)

Ex.4 If \( \log_b x = 1.2 \), \( \log_b y = 3.1 \), \( \log_b z = 11.1 \), evaluate \( \log_b \left( \frac{x}{y} \right) - \log_b (z^2 x) \)
4.5 Logarithmic and Exponential Equations

Strategies to Solve Equis:

**Logarithmic**

1. Get logs all on one side of eqn
2. Condense using log properties
3. Use defn of log to rewrite in exponential form
4. Continue solving
5. Check all answers.
   (This must happen because of restricted domain.)

**Exponential**

1. Isolate exponential
2. Use defn of log to rewrite as log eqn
3. Continue solving

(As you don't have to check answers because there is no restriction on domain.)

Ex 1 Solve: \(4^{x+2} = 64\)
4.5 (cont)

Ex 2. Solve.

(a) \(2e^x + 3 = 13\)

(b) \(5^{x+b} - 4 = 12\)

c) \(\ln (2x-3) = \ln 11\)

d) \(2 \log_4 x = 5\)
Ex 3  Solve.

(a) \( \log_3 (2x) - \log_3 (x-3) = 1 \)

(b) \( 3^{2x} + 3^x = 20 \)

(c) \( \log (x^2) = (\log x)^2 \)
Ex1 If $1000 is invested at 10% compounded continuously, the future value $S$ at any time $t$ (in years) is given by $S = 1000e^{0.1t}$.

(a) What is the account worth after one year?
(b) How long will it take for the investment to double?
Ex 2  The population of Mathville grows according to the formula \( P = P_0 e^{0.03t} \). If the population was 250,000 in the year 2000, estimate the year in which the population reaches 350,000.
4.6 (cont)

Ex. 3 Radioactive Iodine-131 has a half-life of 8 days. How long does it take to reduce an initial amount of Iodine-131 to 1% of the initial amount?

Ex. 4 The tsunami of 2004 killed over 200,000 people and was measured at $M = 9.1$ on the Richter scale. What was its intensity? (Use $M = \log \left( \frac{I}{I_0} \right)$ where $I_0 = 10^{-3}$ is the zero-level earthquake, or the minimum intensity that can be felt.)