3d Geometry Jeopardy

Polyhedra
10 points -- Why can't we make a Platonic solid with hexagonal faces?
   because 3 regular hexagons lie flat and cannot form a dihedral angle

20 points -- Draw a right triangular pyramid.

   (a pyramid w/ equilateral base)

30 points -- List the Platonic Solids.
   cube
dodecahedron
tetraheiran
icosahedron
octahedron

40 points -- A prism has 96 edges. How many vertices and faces does it have?

\[
F + V - 2 = E \\
F + V - 2 = 96
\]

\[
=) 2n + 2n - 2 = 96 \\
3n = 96 \\
\]

\[
\therefore n = 32 \Rightarrow F = 34 \\
V = 64
\]
**Surface Area**

10 points-- Find the surface area of the following solid.

\[ A_\Delta = \frac{1}{2} (5) (2) = 5 \, \text{in}^2 \]

\[ \text{SA} = 2(5) + (6\sqrt{5} + 2\sqrt{5} + 5) \]

\[ = 10(6 + 3\sqrt{5}) \, \text{in}^2 \]

20 points-- Find the surface area of a sphere with radius of 5 meters.

\[ \text{SA} = 4\pi r^2 = 4\pi (25) = 100\pi \, \text{m}^2 \]

30 points-- Find the surface area of a right circular cone with radius of 2 ft. and height of 6 ft.

\[ l = \sqrt{40} = 2\sqrt{10} \]

\[ \text{SA} = \pi (2^2) + \frac{1}{2}(2\pi(2))2\sqrt{10} \]

\[ = 4\pi + 4\sqrt{10}\pi = 4\pi (1 + \sqrt{10}) \, \text{ft}^2 \]
Surface Area (continued)
40 points-- Find the surface area of the following shell.

\[ SA = 300\pi + 240\pi + 316\pi = 576\pi \text{ cm}^2 \]

50 points-- Find the surface area of the following solid.

\[ SA = 15\pi + 60\pi + 9\pi = 84\pi \text{ ft}^2 \]
Volume
10 points-- Find the volume of the following solid.

\[ V = \frac{1}{2} \times (5 \times 2) \times 10 \]
\[ = 50 \text{ in}^3 \]

20 points-- Find the volume of a sphere with radius of 5 meters.

\[ V = \frac{4}{3} \pi (5^3) = \frac{500 \pi}{3} \text{ m}^3 \]

30 points-- Find the volume of a right circular cone with radius of 2 ft. and height of 6 ft.

\[ V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi (2^2)(6) \]
\[ = 8 \pi \text{ ft}^3 \]
Volume (continued)

40 points -- Find the volume of the following shell.

\[ V = V_{\text{outer}} - V_{\text{inner}} \]
\[ = \pi (10^2)(15) - \pi (8^2)(15) \]
\[ 15 \text{ cm} = 540\pi \text{ cm}^3 \]

50 points -- Find the volume of the following solid.

\[ V = \frac{1}{3}\pi (3^2)(4) + \pi (3^2)(10) \]
\[ = 12\pi + 90\pi \]
\[ = 102\pi \text{ ft}^3 \]
Scaling
10 points-- If a cube's sides double in length, what happens to its surface area?

its multiplied by \(2^2 = 4\)

20 points-- If a cube's sides triple in length, what happens to its volume?

its multiplied by \(3^3 = 27\)

30 points-- We have a scale model prism whose height is 5 inches, and we want the actual prism to have a height of 10 feet. What is the relationship between the scale model's surface area and the actual surface area?

\[
5 \text{ in} \rightarrow 10 \text{ ft} = 5 \text{ m} \rightarrow 120 \text{ m}
\]

\[
5S = 120
\]

\[
S = 24
\]

\[
\Rightarrow 5A_{\text{new}} = 24^2 \leq A_{\text{model}}
\]
Scaling (continued)
40 points-- We have a scale model prism whose height is 5 inches, and we want the actual prism to have a height of 10 feet. What is the relationship between the scale model's volume and the actual volume?

\[ V_{\text{actual}} = 24^3 \cdot V_{\text{model}} \]

50 points-- For a right square pyramid with height \( h = 8 \) inches and the base side length = 5 inches, what is the surface area and volume? If we scale that up to have a height of 3 ft, what is its surface area and volume?

\[ h = 8 \text{ in} \]

\[ 8^2 + 2.5^2 = l^2 \]

\[ l^2 = \frac{281}{4} \]

\[ l = \frac{\sqrt{281}}{2} \]

\[ \text{SA} = s^2 + \frac{1}{2}(20)(\frac{\sqrt{281}}{2}) \]

\[ = 25 + 5\sqrt{281} \]

\[ = 108.8 \text{ in}^2 \]

\[ V = \frac{1}{3}Ah = \frac{1}{3}(25)(8) = \frac{200}{3} \]

\[ = 66.67 \text{ in}^3 \]
Hodge Podge
10 points-- Give an exact definition of a sphere.

the set of points equidistant from
a fixed pt (called the center) in 3d

20 points-- What is Euler's Formula and what does it apply to?

\[ F + V - 2 = E \]

all polyhedra

30 points-- Find the area of a regular hexagon whose sides are 4 cm in length.

\[ A_0 = 6 \left( \frac{\sqrt{3}}{4} x^2 \right) = \frac{3\sqrt{3}}{2} x^2 \]

\[ \Rightarrow A = \frac{3\sqrt{3}}{2} \left( 4^2 \right) = 24\sqrt{3} \text{ cm}^2 \]
Hodge Podge (continued)
40 points-- What is the relationship between \( h \), \( l \) and \( e \) (as drawn on this pyramid)?

\[ h^2 + \left(\frac{1}{2} s\right)^2 = l^2 \]

\[ l^2 + \left(\frac{1}{2} s\right)^2 = e^2 \]

50 points-- State the Pythagorean Theorem and give a proof.

In a right triangle, the sum of the lengths of the legs squared is equal to the hypotenuse length squared.

\[ a^2 + b^2 = c^2 \]