6.3 (cont)

Since $D_x[e^x]=e^x$, then $\boxed{\int e^x\,dx = e^x + C}$

Ex 4 Evaluate integrals.

(a) $\int e^{-6x}\,dx$

$u=-6x$
$du=-6\,dx$
$\frac{-1}{6}du=dx$

$=\int e^u\left(\frac{-1}{6}\right)du = \frac{-1}{6}\int e^u\,du$
$=\frac{-1}{6}e^u+C$
$=\frac{-1}{6}e^{-6x}+C$

(b) $\int e^{x+e^x}\,dx = \int \underbrace{e^x e^{e^x}\,dx}_{du}$

let $u=e^x$
$du=e^x\,dx$

$=\int e^u\,du$
$=e^u+C = e^{e^x}+C$

(c) $\int_1^2 \frac{e^{3/x}}{x^2}\,dx$

let $u=\frac{3}{x}=3x^{-1}$
$du=\frac{-3}{x^2}dx$
$\frac{-1}{3}du=\frac{1}{x^2}dx$

if $x=1$, $u=3$
if $x=2$, $u=3/2$

$=\frac{-1}{3}\int_3^{3/2} e^u\,du$

$=-\frac{1}{3}e^u\Big|_3^{3/2} = \frac{-1}{3}\left(e^{3/2}-e^3\right)$

6.5 (cont)

Ex 1 Population of U.S. was 3.9 million in 1790 + 17.8 million in 1960. If the rate of growth is assumed proportional to the population, what estimate would you give for the population in 2000? (compare your answer w/ actual population of 275 million.)

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let time start being measured in 1790

$\Rightarrow$ in 1790, $t=0$ $\quad P_0 = 3,900,000$

in 1960, $t=170$ $\quad P = 178,000,000$

$P = P_0 e^{rt}$ $\qquad$ at $t = 210$ (in yr 2000)

$P = ?$

$$P = 3,900,000 e^{rt}$$

$$178,000,000 = 3,900,000 e^{r(170)}$$

$$\frac{178,000,000}{3,900,000} = e^{170r}$$

$$\ln\left(\frac{1780}{39}\right) = 170r$$

$$r = \frac{1}{170}\ln\left(\frac{1780}{39}\right) \simeq 0.022475$$

$$\Rightarrow P = 3,900,000 e^{0.0222475t}$$

at $t=210$, $P = 3,900,000 e^{0.0222475(210)}$

$$\simeq \boxed{437,377,629}$$

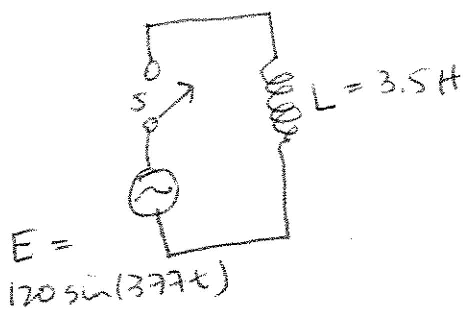
6.6 (cont)

$$\boxed{\text{Kirchoff's Law (for simple electrical circuit)} \quad L\frac{dI}{dt} + RI = E(t)}$$

R = resistance (in ohms Ω)
L = inductance (henrys)
E = voltage (volts)
I = current (amps)

Ex 4 Find I as a function of time for the circuit shown, assuming the switch is closed and $I=0$ at $t=0$.

[$\Rightarrow R=0$ (since there is no resistor)]



Plug in $R=0$, $L=3.5$ and $E = 120\sin(377t)$ into above (boxed) eqn.

$$\Rightarrow 3.5\frac{dI}{dt} + 0 = 120\sin(377t)$$

$$\frac{dI}{dt} = \frac{120}{3.5}\sin(377t)$$

$$\int dI = \int \frac{240}{7}\sin(377t)\,dt$$

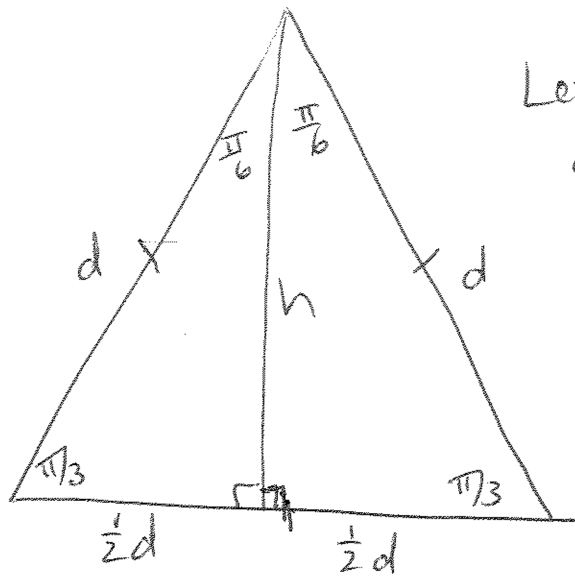
$$I = \frac{240}{7}\left(\frac{-1}{377}\right)\cos(377t) + C$$

but $I=0, t=0$

$$\Rightarrow 0 = \frac{-240}{7(377)}\cos(0) + C$$

$$C = \frac{240}{7(377)} = \frac{240}{2639}$$

$$\Rightarrow \boxed{I = \frac{-240}{2639}\left(\cos(377t) + 1\right)}$$



Let's say we have an equilateral triangle to start with. Drop the $\perp$ height from the top vertex to the base. If each side of the equilateral triangle has length d, then the base is now split into 2 congruent pieces, each of length $\frac{1}{2}d$.

Each angle in an equilateral triangle is the same measure, namely $60°$ or $\pi/3$ radians. The height bisects the top angle $\Rightarrow$ each of those angles now are $\pi/6$.

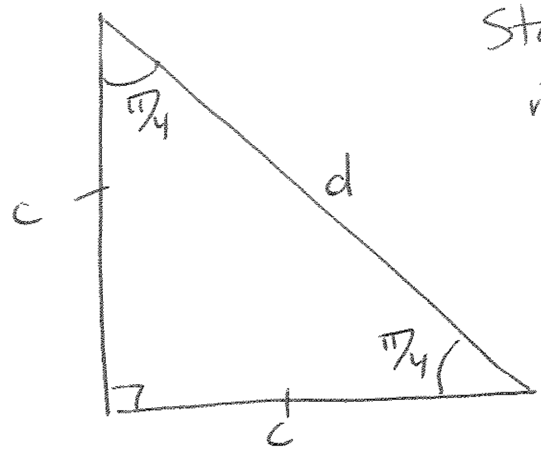
Use Pythagorean Thm to find h (in terms of d). $\Rightarrow$ $h^2 + (\frac{1}{2}d)^2 = d^2$

$$h^2 = d^2 - \frac{1}{4}d^2 = \frac{3}{4}d^2 \Rightarrow h = \frac{\sqrt{3}}{2}d$$

$$\Rightarrow \cos\left(\frac{\pi}{3}\right) = \frac{\frac{1}{2}d}{d} = \frac{1}{2} \qquad \sin\left(\frac{\pi}{3}\right) = \frac{\frac{\sqrt{3}}{2}d}{d} = \frac{\sqrt{3}}{2}$$

and $$\cos\left(\frac{\pi}{6}\right) = \frac{\frac{\sqrt{3}}{2}d}{d} = \frac{\sqrt{3}}{2} \qquad \sin\left(\frac{\pi}{6}\right) = \frac{\frac{1}{2}d}{d} = \frac{1}{2}$$

//



Start w/ an isosceles right triangle ⇒ two legs are congruent and the two acute angles are the same measure, namely $\pi/4$.

Let's call the hypotenuse d and the legs c. Then, using the Pythagorean Thm, we have

$$c^2+c^2=d^2$$
$$2c^2=d^2$$
$$\sqrt{2}\,c=d$$

$$\Rightarrow \cos\left(\frac{\pi}{4}\right)=\frac{c}{d}=\frac{c}{\sqrt{2}\,c}=\frac{1}{\sqrt{2}}=\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)=\frac{\sqrt{2}}{2}$$

and $\sin\left(\frac{\pi}{4}\right)=\frac{c}{d}=\frac{\sqrt{2}}{2}$ also.