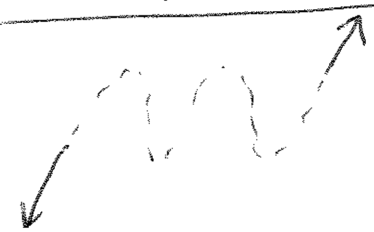
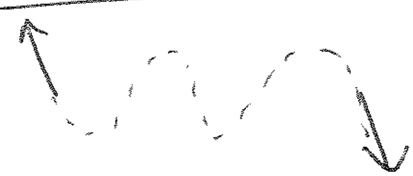
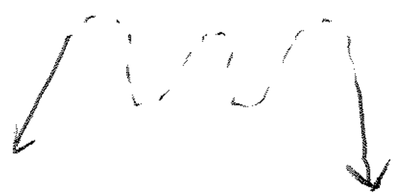
2.2 Polynomial Fns of Higher Degree

polynomial graphs are <u>continuous</u> with <u>smooth</u> rounded turns

Leading Coefficient Test $f(x) = a_n x^n + a_n x^{n-1} + \dots + a_1 x + a_0$

n odd, $a_n > 0$	n odd, $a_n < 0$
	
n even, $a_n > 0$	n even, $a_n < 0$
	

Real Zeros of Polynomial Fn

Equivalent statements: for $a \in \mathbb{R}$ & $f(x)$ polynomial

1. $x = a$ is a zero of $f(x)$.
2. $x = a$ is a solution of $f(x) = 0$.
3. $(x-a)$ is a factor of $f(x)$
4. $(a, 0)$ is an x-intercept of graph of $f(x)$.

Repeated Zeros

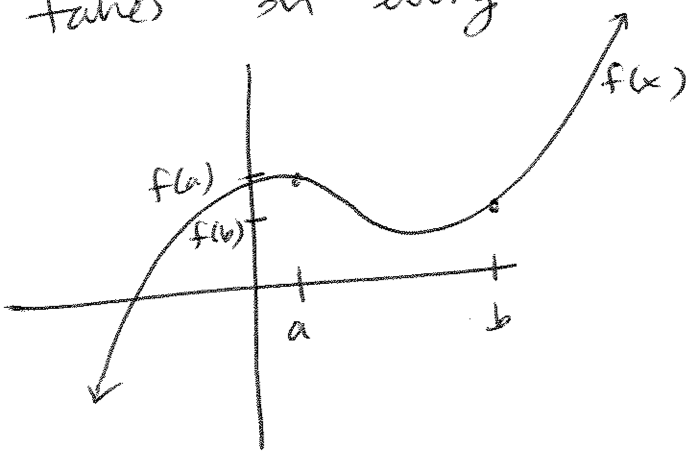
A factor $(x-a)^k$ for $k > 1$ yields a repeated zero $x = a$ of multiplicity k.

1. If k odd, graph crosses x axis at $x = a$.
2. If k even, graph touches x-axis at $x = a$.

2.2 (cont)

Intermediate Value Theorem

Let $a, b \in \mathbb{R} \ni a < b$. If $f(x)$ is a polynomial $\ni f(a) \neq f(b)$, then over x-interval $[a, b]$, f takes on every value between $f(a)$ and $f(b)$.



(✱ we'll use this to help us find zeros.)

Ex 1 ① Describe right-hand and left-hand behavior of the graph. ② Find all zeros ($\mathbb{R}$) w/ its multiplicity. ③ Number of turning points (or humps) on graph?

(a) $f(x) = x^5 + x^3 - 6x$

2.2 (cont)

Ex 1 (cont) (b) $f(x) = 49 - x^2$

Ex 2 Find a polynomial of degree 5 that has zeros $x = -3, 1, 5, 6$.

2.2 (cont)

Ex 3 Sketch the graph of f(x) by looking at the leading coefficient, finding the zeros, and perhaps plotting more pts.

$$f(x) = -48x^2 + 3x^4$$

2.3 Polynomial & Synthetic Division

Factor Theorem

A polynomial $f(x)$ has a factor $(x-k)$ iff

$$f(k)=0.$$

Remainder Theorem

If a polynomial $f(x)$ is divided by $(x-k)$, the remainder is

$$r=f(k).$$

Long Division ⇒ **ALWAYS** useful; always works

Synthetic Division ⇒ **ONLY** useful when dividing by $(x-k)$ $k \in \mathbb{R}$.

Ex 1 Use long division $(5x^3+10x^2-2x+1) \div (2x^2-1)$

2.3 (cont)

Ex 2 Divide $(x^3+4x^2-3x-12) \div (x-3)$

Long Division	Synthetic Division
$x-3 \overline{) x^3+4x^2-3x-12}$	$3 \,\vert\; 1 \quad 4 \quad -3 \quad -12$

Ex 3 Use synthetic division to divide

$(5x^3+6x+8) \div (x+2)$

2.3 (cont)

Ex 4 Write the function as $f(x) = (x-k)\,q(x) + r(x)$
($q(x)$ = quotient, $r(x)$ = remainder)

$f(x) = x^3 - 5x^2 - 11x + 8 \qquad k = -2$

Ex5 Use division to show that $2/3$ is a solution
of $\quad 48x^3 - 80x^2 + 41x - 6 = 0$. Use the result
to factor polynomial completely + find
<u>all</u> solutions.

2.4 Complex Numbers

Defn $a+bi$, $a,b \in \mathbb{R}$, $i=\sqrt{-1}$, is a complex #, with $\mathbb{R}$ part a and imaginary part b.

Properties of $\mathbb{C}$ #s

1. $a+bi = c+di$ iff $a=c$ and $b=d$
2. $\mathbb{R} \subset \mathbb{C}$
3. $(a+bi)+(c+di) = (a+c)+i(b+d)$
4. $(a+bi)-(c+di) = (a-c)+i(b-d)$
5. $(a+bi)(c+di) = ac+adi+bci+bdi^2$ (but $i^2=-1$)
   $=(ac-bd)+(ad+bc)i$
6. $\dfrac{a+bi}{c+di} = \dfrac{(a+bi)}{(c+di)}\left(\dfrac{c-di}{c-di}\right) = \dfrac{ac-adi+bci-bdi^2}{c^2-d^2i^2}$
   $$= \frac{(ac+bd)+i(bc-ad)}{c^2+d^2} = \left[\frac{ac+bd}{c^2+d^2}\right] + i\left[\frac{bc-ad}{c^2+d^2}\right]$$
7. complex conjugate of $a+bi$ is $a-bi$.

Principal Square Root of Negative

$a \in \mathbb{R}$, $a>0$, $\sqrt{-a} = \sqrt{a}\, i$

$i^2 = -1$

$i^3 = i^2 i = -i$

$i^4 = i^2 i^2 = -1(-1) = 1$

$i^5 = i^4 \cdot i = i$

$i^6 = i^4 \cdot i^2 = -1$

$i^7 = i^4 \cdot i^3 = -i$

$i^8 = i^4 \cdot i^4 = 1$

$i^9 = i^8 \cdot i = i$

etc.

2.4 (cont)

Ex 1 write in standard form: $-4i^2+2i$

Ex 2 Perform operation & write in standard form.

(a) $\dfrac{6-7i}{1-2i}$

(b) $(1-2i)^2-(1+2i)^2$

2.4 (cont)

Ex 3 Write in standard form.

$$(2-\sqrt{-6})^2$$

Ex 4 Use quadratic formula to solve

$$16x^2-4x+3=0$$