2.2 Polynomial Fns of Higher Degree

Polynomial graphs are continuous with smooth rounded turns.

Leading Coefficient Test: \( f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0 \)

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<th>( n ) odd, ( a_n &gt; 0 )</th>
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<td>![Wavy Graph]</td>
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Real Zeros of Polynomial Fn

Equivalent statements: for \( a \in \mathbb{R} \) and \( f(x) \) polynomial

1. \( x = a \) is a zero of \( f(x) \).
2. \( x = a \) is a solution of \( f(x) = 0 \).
3. \((x-a)\) is a factor of \( f(x) \).
4. \((a,0)\) is an \( x \)-intercept of graph of \( f(x) \).

Repeated Zeros

A factor \((x-a)^k\) for \( k > 1 \) yields a repeated zero \( x = a \) of multiplicity \( k \).

- If \( k \) odd, graph crosses \( x \)-axis at \( x = a \).
- If \( k \) even, graph touches \( x \)-axis at \( x = a \).
Intermediate Value Theorem

Let $a, b \in \mathbb{R}$ with $a < b$. If $f(x)$ is a polynomial and $f(a) \neq f(b)$, then over the interval $[a, b]$, $f$ takes on every value between $f(a)$ and $f(b)$.

We'll use this to help us find zeros.

Example

1. Describe right-hand and left-hand behavior of the graph.
2. Find all zeros (IR) with its multiplicity.
3. Number of turning points (or bumps) on graph?

(a) $f(x) = x^5 + x^3 - 6x$
2.2 (cont)
Ex 1 (cont) (b) \( f(x) = 49 - x^2 \)

Ex 2 Find a polynomial of degree 5 that has zeros \( x = -3, 1, 5, 6 \).
Ex 3 Sketch the graph of \( f(x) \) by looking at the leading coefficient, finding the zeros, and perhaps plotting more pts.

\[
f(x) = -48x^5 + 3x^4
\]
2.3 Polynomial & Synthetic Division

Factored Theorem
A polynomial \( f(x) \) has a factor \( (x-k) \) iff
\[ f(k) = 0. \]

Remainder Theorem
If a polynomial \( f(x) \) is divided by \( (x-k) \), the remainder is
\[ r = f(k). \]

Long Division =) **Always** useful; always works

Synthetic Division =) **ONLY** useful when dividing by \( (x-k) \) \( k \in \mathbb{R} \).

Ex 1. Use long division \((5x^3 + 10x^2 - 2x + 1) \div (2x^2 - 1)\)
Ex 2. Divide \((x^3 + 4x^2 - 3x - 12) \div (x - 3)\)

**Long Division**

\[
x - 3 \overline{\bigg| x^3 + 4x^2 - 3x - 12}
\]

**Synthetic Division**

\[
\begin{array}{c|cccc}
3 & 1 & 4 & -3 & -12 \\
\end{array}
\]

Ex 3. Use synthetic division to divide \((5x^3 + 6x + 8) \div (x + 2)\)
Ex 4 Write the function as \( f(x) = (x-k)q(x) + r(x) \)  
(\( q(x) \) = quotient, \( r(x) \) = remainder)  
\( f(x) = x^3 - 5x^2 - 11x + 8 \) \( k = -2 \)

Ex 5 Use division to show that \( \frac{3}{2} \) is a solution of \( 48x^3 - 80x^2 + 41x - 6 = 0 \). Use the result to factor polynomials completely and find all solutions.
2.4 Complex Numbers

**Defn** \( a+bi, a, b \in \mathbb{R}, i = \sqrt{-1}, \) is a complex 
#, with \( \mathbb{R} \) part \( a \) and imaginary part \( b \).

**Properties of C #s**

1. \( a+bi = c+di \) if \( a=c \) and \( b=d \)
2. \( \mathbb{R} \subset \mathbb{C} \)
3. \((a+bi) + (c+di) = (a+c) + i(b+d)\)
4. \((a+bi) - (c+di) = (a-c) + i(b-d)\)
5. \((a+bi)(c+di) = ac + adi + bci + bd i^2 \quad (bd \ i^2 = -1)\)
   \[= (ac - bd) + (ad + bc)i\]
6. \( a+bi = \frac{(a+bi)}{(c+di)} \frac{(c-di)}{(c-di)} = \frac{ac-ad+i(bc-ad)}{c^2-d^2} \)
   \[= \frac{(ac+bd)}{c^2+d^2} + i\frac{(bc-ad)}{c^2+d^2}\]
7. Complex conjugate of \( a+bi \) is \( a-bi \).

**Principal Square Root of Negative #**

\( a \in \mathbb{R}, \sqrt{-a} = \sqrt{a} i \)

\( a > 0 \)

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<th>( i^n )</th>
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<tbody>
<tr>
<td>1</td>
<td>( i )</td>
</tr>
<tr>
<td>2</td>
<td>( -1 )</td>
</tr>
<tr>
<td>3</td>
<td>( -i )</td>
</tr>
<tr>
<td>4</td>
<td>( 1 )</td>
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<tr>
<td>5</td>
<td>( i )</td>
</tr>
<tr>
<td>6</td>
<td>( 1 )</td>
</tr>
<tr>
<td>7</td>
<td>( i )</td>
</tr>
<tr>
<td>8</td>
<td>( 1 )</td>
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\( i^8 = i^4 \cdot i^4 = 1 \)

\( i^{2n} = 1 \), \( i^{2n+1} = i \) etc.
Ex 1: Write in standard form: $-4i^2 + 2i$

Ex 2: Perform operation and write in standard form.

(a) \( \frac{6 - 7i}{1 - 2i} \)

(b) \((1 - 2i)^2 - (1 + 2i)^2\)
Ex. 3 Write in standard form.
\[(2 - \sqrt{6})^2\]

Ex. 4 Use quadratic formula to solve
\[16x^2 - 4x + 3 = 0\]