1.6 Library of Parent Functions

<table>
<thead>
<tr>
<th>Constant Fn</th>
<th>Identity Fn</th>
<th>Abs. Value Fn</th>
<th>Square Root Fn</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = f(x) = c$</td>
<td>$f(x) = y = x$</td>
<td>$y = f(x) =</td>
<td>x</td>
</tr>
<tr>
<td>Quadratic Fn</td>
<td>Cubic Fn</td>
<td>Reciprocal Fn</td>
<td>Greatest Integer Fn</td>
</tr>
<tr>
<td>$y = f(x) = x^2$</td>
<td>$y = f(x) = x^3$</td>
<td>$y = f(x) = \frac{1}{x}$</td>
<td>$y = f(x) = [x]$</td>
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</tbody>
</table>

General polynomial graphs:
Ex 1 Graph these functions.

(a) \( f(x) = [x - 3] \)

(b) \( y = [x] - 1 \)

(c) \( f(x) = \begin{cases} 1 - (x-1)^2 & , x \leq 2 \\ \frac{1}{\sqrt{x-2}} & , x > 2 \end{cases} \)

(d) \( f(x) = 2(x+3)^2 + 1 \)
Ex 2  Given the graph, write the equation for the function it represents.

(a) 

(b) 

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### 1.7 Transformations of Functions

#### Types of Transformations to \( y = f(x) \)

<table>
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<th>Transformation</th>
<th>Function</th>
<th>Example</th>
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<tbody>
<tr>
<td>1. Shift</td>
<td>( h(x) = f(x) + c ) ( h(x) = f(x-c) )</td>
<td>( y = x^2 + 2 ) ( y = (x-1)^3 )</td>
</tr>
<tr>
<td>2. Reflect</td>
<td>( g(x) = -f(x) ) ( g(x) = f(-x) )</td>
<td>( y = -x^2 ) ( y = \sqrt{-x} )</td>
</tr>
<tr>
<td>3. Stretch/Shrink</td>
<td>( k(x) = cf(x) ) ( k(x) = f(cx) )</td>
<td>( y = 5x^3 ) ( y = \sqrt{\frac{1}{2}}x )</td>
</tr>
</tbody>
</table>

Ex 1. Graph these functions.

(a) \( y = \sqrt{-x} \)

(b) \( y = -x^2 + 3 \)

(c) \( y = |x-2| + 1 \)
Ex 2  Write an eqn for these graphs.

Ex 3  Describe transformations compared to base graph.

(a)  \( g(x) = 2(x+1)^3 - 9 \)

(b)  \( h(x) = -\lfloor x + 4 \rfloor - 3 \)
Ex 4: Given this graph for \( f(x) \), sketch the graphs of the transformed functions.

(a) \( f(-x) \)

(b) \( f(x-1) + 3 \)

(c) \( -2f(x) \)
1.8 Combinations of Functions: Composite Fns

Operations w/ Fns

1. Sum: \((f+g)(x) = f(x) + g(x)\)

2. Difference: \((f-g)(x) = f(x) - g(x)\)

3. Product: \((f \cdot g)(x) = f(x)g(x)\)

4. Quotient: \((f \div g)(x) = \frac{f(x)}{g(x)} , g(x) \neq 0\)

5. Composition: \((f \circ g)(x) = f(g(x))\)

Ex 1 For \(f(x) = \sqrt{x^2-4}\) and \(g(x) = \frac{x^2}{x^2+1}\), find

(a) \((f+g)(x)\)

(c) \((\frac{f}{g})(x)\)

(b) \((fg)(x)\)

(d) \((f-g)(x)\)
Ex 2. For \( f(x) = \sqrt{x^2 - 4} \) and \( g(x) = \frac{x^2}{x^2 + 1} \), find

(a) \( f(g(x)) \) and domain

(b) \( g(f(x)) \) and domain

Ex 3. For \( f(x) = x^3 - 1 \) and \( g(x) = 2x + 5 \), find

(a) \( \left( \frac{f}{g} \right)(0) \)

(b) \( (fg)(2) + g(4) \)

(c) \( f(g(0)) \)
Ex 4. For \( f(x) = 3x + 5 \), find \( (f \circ f)(x) \), and its domain.

Ex 5. Given \( h(x) = \frac{4}{(5x+1)^2} \), find two functions \( f \) and \( g \) such that \( (f \circ g)(x) = h(x) \).
1.9 Inverse Functions

**Defn** If \( f(f^{-1}(x)) = f^{-1}(f(x)) = x \), then \( f(x) \) and \( f^{-1}(x) \) are inverse functions. \( \forall x \in \text{domain} \) (notation: \( f^{-1}(x) \) is read "f inverse of x"; that -1 indicates the inverse, it's not an exponent)

**Vocab**

one-to-one: a fn is one-to-one if each output has exactly one input. (it passes horizontal line test)

A one-to-one fn is invertible!

Finding an Inverse Fn

1. "Pants" method
2. Algebraic Technique
1.9 (cont)

Ex 1 For $f(x)$, find the inverse function $f^{-1}(x)$.

(a) $f(x) = \frac{x^5 - 1}{3}$

(b) $f(x) = \sqrt[3]{x+2} + 1$

Ex 2 Show that $f(x) = \frac{1}{1+x}, x \geq 0$, and $g(x) = \frac{1-x}{x}$, $0 < x \leq 1$ are inverse functions.

(a) algebraically

(b) graphically
Ex 3 Find the inverse function of 

\[ f(x) = \frac{x-3}{x+2} \]. Then, graph both \( f(x) \) and \( f^{-1}(x) \) on same coordinate plane. State domain and range of \( f \) and \( f^{-1} \).
Ex 4 Use $f(x) = 3x + 4$ and $g(x) = x^3$ to find $(g \circ f)^{-1}$. 