Log Properties Proofs

(a) <u>claim</u> $\log_a y^b = b\log_a y$

<u>Pf</u> By definition, $a^x = y \Leftrightarrow \log_a y = x$.

$\Rightarrow$ $(a^x)^b = y^b$ is equivalent to $a^{xb} = y^b$ and

(☆) $\log_a y^b = xb$

But $xb = (\log_a y)b = b\log_a y$

$\Rightarrow$ (☆) becomes $\log_a y^b = b\log_a y$ //

(b) <u>claim</u> $\log_a(mn) = \log_a m + \log_a n$.

<u>Pf</u> Let $a^x = m$ and $a^w = n$.

Then $\log_a m = x$ and $\log_a n = w$ (by defn of log).

$\Rightarrow$ $a^{x+w} = a^x a^w = mn$

$\Rightarrow$ $\log_a(mn) = x+w$ (by defn of log)

but $x+w = \log_a m + \log_a n$

$\Rightarrow$ $\log_a(mn) = \log_a m + \log_a n$. //

(c) <u>claim</u> $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

<u>pf</u> Since we have already proved the first two properties, we can use them here.

$$\Rightarrow \log_a\left(\frac{x}{y}\right) = \log_a\left(x\left(y^{-1}\right)\right)$$

$$= \log_a x + \log_a\left(y^{-1}\right)$$

$$= \log_a x + -1 \cdot \log_a y$$

$$= \log_a x - \log_a y \quad //$$