Math1100 Final Review Problems

From all sections covered in class in chapters 9-14
Fall, 2019

1. Compute the following limits.

(a) \[ \lim_{x \to \infty} \frac{3x^2 + 1}{5x^2 + x + 5} = \lim_{x \to \infty} \frac{3x^2}{5x^2} = \frac{3}{5} \]

(b) \[ \lim_{x \to \infty} \frac{3x + 1}{5x^2 + x + 5} = \lim_{x \to \infty} \frac{3x}{5x^2} = \lim_{x \to \infty} \frac{3}{5x} = 0 \]

(c) \[ \lim_{x \to \infty} \frac{3x^3 + 1}{5x^2 + x + 5} = \lim_{x \to \infty} \frac{3x^3}{5x^2} = \lim_{x \to \infty} \frac{3x}{5} = \infty \text{ or DNE} \]

(d) \[ \lim_{x \to 5} \frac{3x^2 - 6x - 45}{2x^2 - 9x - 5} = \lim_{x \to 5} \frac{(x-5)(3x+9)}{(x-5)(2x+1)} = \lim_{x \to 5} \frac{3x+9}{2x+1} = \frac{24}{11} \]

\[ \frac{0}{0} \text{ case} \]

(e) \[ \lim_{x \to 1} \frac{x^2 + x - 2}{x^2 + 3x - 4} = \lim_{x \to 1} \frac{(x-1)(x+2)}{(x-1)(x+4)} = \lim_{x \to 1} \frac{x+2}{x+4} = \frac{3}{5} \]

\[ \frac{0}{0} \text{ case} \]

(f) \[ \lim_{x \to 2} \frac{x + 9}{x^2 - 4} = \text{DNE} \]

\[ \frac{1}{0} \text{ case} \]

(g) \[ \lim_{x \to 2} \frac{x^2 - 4}{x + 9} = \frac{0}{11} = 0 \]

(h) \[ \lim_{x \to 1} \frac{x^2 + x - 2}{x^2 + 4x - 5} = \lim_{x \to 1} \frac{(x-1)(x+2)}{(x-1)(x+5)} = \lim_{x \to 1} \frac{x+2}{x+5} = \frac{3}{6} = \frac{1}{2} \]

\[ \frac{0}{0} \text{ case} \]


(i) \[ \lim_{x \to \infty} \frac{e^{x^2} - 2x^3}{14x - 3x^3} = \lim_{x \to \infty} -\frac{2x}{-3x^3} = \frac{2}{3} \]

2. Find the derivative, \( \frac{dy}{dx} \) of the following functions.

(a) \( y = x^4 - 5x^3 + 7 - 3x + \ln x \)
\( \frac{dy}{dx} = y' = 4x^3 + 15x^2 - 3x(\ln x) + \frac{1}{x} \)

(b) \( y = \frac{x^2 - 6x + 9}{\ln x + 5x} \)
\( \frac{dy}{dx} = y' = \frac{(\ln x + 5x)(2x - 6) + (x^2 - 6x + 9)(\frac{1}{x} + 5)}{(\ln x + 5x)^2} \)

(c) \( y = (x - 7x^3)^9(e^{x^3 + 3x}) \)
\( y' = (x - 7x^3)^9(e^{x^3 + 3x}(2x + 3)) + 9(x - 7x^3)^8(1 - 3x^4)(e^{x^3 + 3x}) \)

(d) \( \sqrt{x^2 - 9x} = e^{y} + \frac{1}{5}x^2 \)
\( \frac{dy}{dx} = \frac{2x - 9}{2\sqrt{x^2 - 9x}} - \frac{2}{5}x \)

(e) \( x^3 + 8 = \ln(xy) \)
\( \frac{dy}{dx} = \frac{3x^2 - x}{x(y + x\frac{dy}{dx})} \)

(f) \( y = \sqrt{5x + \ln(x^2 + x^7)} \)
\( \frac{dy}{dx} = \frac{3x^3 y - y^3}{x^2 + x^7} \)

(g) \( y = 3e^{x} + e^{-x} + x^4e^{x} \)
\( y' = 3x^4(\ln 3)(4x^3) + e^{x^4}(4x^3) + 4e^x e^{-1} \)

(h) \( y = \frac{(7x^4 + e^x + 2x)^7}{\sqrt{2x + x^5}} \)
\( y' = \frac{3(2x + x^5)^2}{(7x^4 + e^x + 2x)^6} \)

\( 28x^3 + e^x(3x^2 + 2) - (7x^4 + e^x + 2x)^7 \left( \frac{1}{2}(2x + x^5)^2 \right) \)
3. Find the equation of the tangent line for the given curve at the indicated point.

(a) \( x^3 + xy + 4 = 0 \) at \((2, -6)\)

\[
\text{derivative: } 3x^2 + y + x y' = 0 \Rightarrow y' = \frac{-3x^2 - y}{x}
\]

\[
m = \frac{-3}{2}, \quad (2, -6)
\]

line: \( y + 6 = -\frac{3}{2}(x - 2) \) \(\Rightarrow\) \( y = -3x \)

(b) \((x + 2y)e^{xy} = xy^2\) at \((0, 0)\)

\[
\text{derivative: } (1+2y')e^{xy} + (x+2y)e^{xy}(y + xy') = y^2 + x(2y)y'
\]

plug in \((0,0)\) to derivative: \((1+2y')e^{0} + (0+0)e^{0}(0+0) = 0+0
\]

\[
\Rightarrow 1+2y' = 0 \quad \Rightarrow y' = -\frac{1}{2} = \text{slope}
\]

pt \((0,0)\), \(m = \frac{-1}{2} \)

line: \( y - 0 = -\frac{1}{2}(x-0) \) \(\Rightarrow\) \( y = -\frac{1}{2}x \)

(c) \(3x^2 - 4xy + 2y^3 = 2x + 16\) at \((0, 2)\)

\[
\text{derivative: } 6x(4y+4xy') + 6y^2y' = 2
\]

plug in \((0,2)\): \(6(0) - (4(2) + 0(y')) + 6(2)^3y' = 2
\]

\[\Rightarrow -8 + 24y' = 2 \quad \Rightarrow y' = \frac{5}{12} = \text{slope}\]

pt \((0,2)\), slope \(m = \frac{5}{12} \)

\(\Rightarrow\) line: \( y = \frac{5}{12}x + 2 \)

(d) \(f(x) = x^2 - 3x\) at \(x = 2\)

\[
\text{derivative: } f'(x) = 2x - 3 \quad \text{slope: } \quad f'(2) = 2(2) - 3 = 1 = m
\]

pt: \((2, -2)\), \(f(2) = 2^2 - 3(2) = -2
\]

\(\Rightarrow\) line is \( y - (-2) = 1(x-2) \) \(\Rightarrow\) \( y = x - 4 \)

(e) \(f(x) = x^2 - 3x\) at \(x = 1\)

(same fn as (d)) \( f'(x) = 2x - 3
\]

\[m = \text{slope} = f'(1) = 2(1) - 3 = -1\]

pt: \((1, -2)\), \(f(1) = 1^2 - 3(1) = -2
\]

\(\Rightarrow\) line is \( y + 2 = -1(x-1) \) \(\Rightarrow\) \( y = -x - 1 \)
4. Find \( f^{(4)}(x) \) if \( f(x) = \frac{1}{x^2} + 7x^3 - e^x + 5x \).
\[
\begin{align*}
  f'(x) &= -2x^{-3} + 21x^2 - e^x + 5 \\
  f''(x) &= 6x^{-4} + 42x - e^x \\
  f'''(x) &= -24x^{-5} + 42 - e^x \\
  f^{(4)}(x) &= 120x^{-6} - e^x
\end{align*}
\]

5. Find the area between the two given curves.

(a) \( y = 6 - x^2 \) and \( y = x \)

\[
\text{Intersection points:} \\
\begin{align*} 
  x &= 6 - x^2 \\
  x^2 + x - 6 &= 0 \\
  (x+3)(x-2) &= 0 \\
  x &= -3, 2
\end{align*}
\]
\[
A = \int_{-3}^{2} ((6 - x^2) - x) \, dx = \left[ (6x - \frac{x^3}{3} - \frac{x^2}{2}) \right]_{-3}^{2} = (6(2) - \frac{8}{3} - 2) - (-18 + 9 - \frac{9}{2}) = \frac{125}{6}
\]

(b) \( y = 4x + 3 \) and \( y = x^2 + 3 \)

\[
\text{Intersection point:} \\
4x + 3 = x^2 + 3 \\
\begin{align*} 
  x^2 - 4x &= 0 \\
  x(x-4) &= 0 \\
  x &= 0, 4
\end{align*}
\]
\[
A = \int_{0}^{4} (4x+3-(x^2+3)) \, dx = \int_{0}^{4} (4x-x^2) \, dx = \left[ 2x^2 - \frac{x^3}{3} \right]_{0}^{4} = 2(16) - \frac{64}{3} - 0 = \frac{32}{3}
\]

(c) \( y = 3x + 2 \) and \( y = x^2 + 2 \)

\[
\text{Intersection point:} \\
3x + 2 = x^2 + 2 \\
\begin{align*} 
  3x &= x^2 \\
  x^2 - 3x &= 0 \\
  x(x-3) &= 0 \\
  x &= 0, 3
\end{align*}
\]
\[
A = \int_{0}^{3} (3x+2-(x^2+2)) \, dx = \int_{0}^{3} (3x-x^2) \, dx = \left[ \frac{3x^2}{2} - \frac{x^3}{3} \right]_{0}^{3} = (\frac{27}{2} - 9) - 0 = \frac{9}{2}
\]
5. Find the area between the two given curves.
(d) \( y = 8 - x^2 \) and \( y = x^2 \)

\[
A = \int_{-2}^{2} (8 - x^2 - x^2) \, dx = \int_{-2}^{2} (8 - 2x^2) \, dx = \left[ 8x - \frac{2}{3}x^3 \right]_{-2}^{2} = \left( 16 - \frac{16}{3} \right) - \left( -16 + \frac{16}{3} \right) = 32 - \frac{32}{3} = \frac{64}{3}
\]

6. Given the function \( f(x, y) = \frac{7x - 4y^2}{\sqrt{5x}} \)
(a) State the domain of the function.
\[ x > 0, \ y \in \mathbb{R} \]
(b) Evaluate the function at \((2,1)\).
\[
f(2,1) = \frac{7(2) - 4(1^2)}{\sqrt{5(2)}} = \frac{14 - 4}{\sqrt{10}} = \frac{10}{\sqrt{10}} = \frac{10\sqrt{10}}{10} = \sqrt{10}
\]

7. The cost of producing \( x \) microwave ovens is \( C(x) = 0.01x^2 + 20x + 300 \) dollars, and the revenue function for the product is \( R(x) = 164x \).
(a) What is the profit function?
\[
\pi = R - C = 164x - (0.01x^2 + 20x + 300) = -0.01x^2 + 144x - 300 = \pi(x)
\]
(b) How many microwave ovens should be sold to maximize profit?
\[
\pi'(x) = -0.02x + 144 = 0
\]
\[
x = \frac{144}{0.02} = \frac{14400}{2} = 7200
\]
(c) What is the maximum profit?
\[
\pi(7200) = -0.01(7200^2) + 144(7200) - 300 = \$518100
\]
8. Compute the following integrals.

(a) \( \int (2x^2 - x^4 - 5x^3 + 9) \, dx \)
\[ = \frac{2x^3}{3} - \frac{x^5}{5} - \frac{5x^4}{4} + 9x + C \]

(b) \( \int \left( \frac{5}{x^2} + e^x - \frac{2}{x} \right) \, dx \)
\[ = \left( \frac{5}{x} + e^x - 2 \ln |x| \right) + C = \frac{5}{x} + e^x - 2 \ln |x| + C \]

(c) \( \int 3xe^{x^2+5} \, dx \)
\[ = 3 \left( \frac{1}{2} \right) \int e^u \, du \]
\[ u = x^2 + 5 \]
\[ du = 2x \, dx \]
\[ \frac{1}{2} \, du = x \, dx \]
\[ = \frac{3}{2} e^{x+5} + C \]

(d) \( \int \frac{x^3 + 4x - x^{-1}}{x} \, dx \)
\[ = \int (x^2 + 4 - x^{-2}) \, dx = \frac{x^3}{3} + 4x - \frac{x^{-1}}{-1} + C \]
\[ = \frac{1}{3}x^3 + 4x + \frac{1}{x} + C \]

(e) \( \int 100e^{-0.5x} \, dx \)
\[ u = -0.5x \]
\[ du = -0.5 \, dx \]
\[ -2du = dx \]
\[ = -200 \int e^u \, du \]
\[ = -200e^{-0.5x} + C \]

(f) \( \int (3x^2 - 8x + 2)^9(3x - 4) \, dx \)
\[ u = 3x^2 - 8x + 2 \]
\[ du = (6x - 8) \, dx \]
\[ \frac{1}{2} \, du = (3x - 4) \, dx \]
\[ = \frac{1}{2} \int u^9 \, du \]
\[ = \frac{1}{2} \left( \frac{u^{10}}{10} \right) + C \]
\[ = \frac{1}{20} (3x^2 - 8x + 2)^{10} + C \]
(g) \( \int \frac{2x^2}{x^3-1} \, dx \) \\
\[ u = x^3 - 1 \] \\
\[ du = 3x^2 \, dx \] \\
\[ \frac{1}{3} \, du = x^2 \, dx \]

\[ \int \frac{1}{u} \, du = \frac{2}{3} \ln |u| + C \]

\[ = \frac{2}{3} \ln |x^3 - 1| + C \]

(h) \( \int \frac{-4}{2x-5} \, dx \) \\
\[ u = 2x - 5 \] \\
\[ du = 2 \, dx \] \\
\[ -2 \, du = 4 \, dx \]

\[ = -2 \int \frac{1}{u} \, du = -2 \ln |u| + C \]

\[ = -2 \ln |2x - 5| + C \]

(i) \( \int (4x - 6x^2) \, dx \)

\[ = (2x^2 - 2x^3) \bigg|_1^3 = (2(9) - 2(27)) - (2(1) - 2(1)) \]

\[ = 18 - 54 - 0 = -36 \]

(j) \( \int \left( 3x^3 + 2x - \frac{5}{x^2} \right) \, dx \)

\[ = \left( \frac{3x^4}{4} + x^2 - \frac{5}{x} \right) \bigg|_1^5 \]

\[ = \left( \frac{3}{4} - 1 + \frac{5}{x} \right) \bigg|_1^5 \]

\[ = \left( \frac{3}{4} (625) + 25 + 1 \right) - \left( \frac{3}{4} + 1 + 5 \right) = 488 \]

(k) \( \int \left( 6x^2 + x - \frac{5}{x} \right) \, dx \)

\[ = \left( 2x^3 - \frac{x^2}{2} + \frac{5}{x} \right) \bigg|_1^4 \]

\[ = (2(64) - \frac{16}{2} + \frac{5}{4}) - (2 - \frac{1}{2} + 5) \]

\[ = 114.75 \]
(l) \[ \int_1^2 (4x^3 + 5x - \frac{6}{x^7}) \, dx = \int_1^2 (4x^3 + 5x - 6x^{-3}) \, dx \]
\[= [(x^4 + \frac{5}{2}x^2 + \frac{3}{x^2})]_1^2 \]
\[= (16 + \frac{5}{2}(4) + \frac{3}{4}) - (1 + \frac{5}{2} + 3) = 20.25\]

(m) \[ \int_3^3 \ln x \, dx = 0 \]
(you don't need to do this integral; it asks for area under curve with 0 width, i.e. from \(x=3\) to \(x=3\) \(\Rightarrow\) area is 0)

(n) \[ \int_0^3 x(8x^2 + 9)^{-1/2} \, dx \]
\[u = 8x^2 + 9 \]
\[du = 16x \, dx \]
\[\frac{1}{16} \int_{u(0)}^{u(3)} u^{-1/2} \, du \]
\[= \frac{1}{16} \left( \frac{u^{1/2}}{1/2} \right) \bigg|_0^3 \]
\[= \frac{1}{8} \frac{8x^2+9}{9} \bigg|_0^3 = \frac{1}{8} \sqrt{8x^2+9} \bigg|_0^3 = \frac{3}{2} \]

9. For the function \(y = x^3 - 2x^2 + x + 1\), answer the following questions.

(a) Find the horizontal and vertical asymptotes, if there are any.
\[\text{none}\]

(b) Fill in the first derivative sign line and find the min/max points.
\[y' = 3x^2 - 4x + 1 = (3x-1)(x-1) = 0 \quad (\Rightarrow \quad x = \frac{1}{3}, 1)\]
\[\text{min pt: } (\frac{1}{3}, 1) \quad \text{max pt: } (1, 2) \quad \text{min pt: } (\frac{1}{3}, 1) \quad \text{max pt: } (1, 2)\]

(c) Fill in the second derivative sign line and find the inflection points.
\[y'' = 6x-4 = 0 \quad (\Rightarrow \quad x = \frac{2}{3}) \quad y(\frac{2}{3}) = \frac{\frac{4}{3} - \frac{8}{3} + \frac{2}{3} + 1}{2} = \frac{25}{27}\]
\[\text{inflexion pt: } (\frac{2}{3}, \frac{25}{27})\]

(d) Sketch the graph of this function, given all the answers for questions (a) through (c).
10. For the function \( y = x^4 - 2x^3 + x^2 \), answer the following questions.
   (a) Find the horizontal and vertical asymptotes, if there are any.

   (b) Fill in the first derivative sign line and find the min/max points.

\[
y' = 4x^3 - 6x^2 + 2x = 2x(2x^2 - 3x + 1) = 2x(2x-1)(x-1) = 0 \quad x = 0, \frac{1}{2}, 1
\]

\[\text{min pt: } (0, 0), (1, 0)\]

\[\text{max pt: } \left(\frac{1}{2}, \frac{1}{16}\right)\]

(c) Fill in the second derivative sign line and find the inflection points.

\[
y'' = 12x^2 - 12x + 2 = 2(6x^2 - 6x + 1) = 0 \quad x = \frac{6 \pm \sqrt{36 - 4(6)}}{12} \approx 0.79, 0.21
\]

\[\text{inflection pts: } (0.79, 0.03) \& (0.21, 0.03)\]

(d) Sketch the graph of this function, given all the answers for questions (a) through (c).

11. For the function \( f(x) = \frac{x^2 - 2x + 5}{(x-3)^2} \) with \( f'(x) = \frac{-4(x+1)}{(x-3)^3} \) and \( f''(x) = \frac{14x+27}{(x-3)^4} \), answer the following questions.

   (a) Find the horizontal and vertical asymptotes, if there are any.

   \[\text{VA: } x=3 \quad \text{HA: } y=1\]

   \[\lim_{x \to \infty} \frac{x^2-2x+5}{(x-3)^2} = \lim_{x \to \infty} \frac{x^2}{x^2} = 1\]

   (b) Fill in the first derivative sign line and find the min/max points.

   \[-\frac{4(x+1)}{(x-3)^3} = 0 \text{ when } x=-1 \quad \text{also } x=3 \text{ makes derivative undefined}\]

\[\text{min pt: } (-1, \frac{1}{2}) \quad f(-1) = \frac{1+2+5}{14} = \frac{1}{2}\]

\[\text{no max pt.}\]

(c) Fill in the second derivative sign line and find the inflection points.

\[\frac{14x+27}{(x-3)^4} = 0 \text{ when } x=-\frac{27}{14}\]

\[\text{and } f''(x) \text{ undefined @ } x=3\]

\[\text{inflection pt: } \left(-\frac{27}{14}, 0.52\right) \approx (-1.93, 0.52)\]

(d) Sketch the graph of this function, given all the answers for questions (a) through (c).
12. Suppose the revenue of a company can be modeled by the function \( R(x) = 32x - 0.05x^2 \) where \( R(x) \) is the revenue in thousands of dollars from the sale of \( x \) thousand units of products.

(a) Find the marginal revenue function, \( MR(x) \).

(b) How many units should be sold to maximize revenue?

\[
\begin{align*}
32 - 0.1x &= 0 \\
0.1x &= 32 \\
x &= 320 \text{ thousand units}
\end{align*}
\]

(c) What is the maximum revenue?

\[
R(320) = 32(320) - 0.05(320^2) = 5120 \text{ thousand dollars (i.e. } \$5120000)
\]

(d) If the production is limited to 250 units, how many units will maximize the total revenue?

\[
250,000
\]

(e) Write in words what \( MR(10) \) means.

13. A farmer has 200 feet of fencing and wishes to construct two pens for his animals by first building a fence around a rectangular region, and then subdividing that region into two smaller rectangles by placing a fence parallel to one of the sides. What dimensions of the region will maximize the total area? 

\[
A = xy \\ x, y = ? \text{ to maximize Area} \\
A' = x + y \\
A' = \frac{200}{3} - \frac{y}{3}x = 0 \ (\Rightarrow) \ x = 50 \\
(\Rightarrow) \ \text{max area when } x = 50 \text{ ft} \\
= y = \frac{200 - 2(50)}{3} \ \\
\]

\[
\text{This is concave down parabola} \\
\text{Vertex is max pt.}
\]
14. For the function \( g(x, y, z) = x^2 ye^z \) find \( \frac{\partial g}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial g}{\partial z} = 2x ye^z + x^2 e^z + x^2 ye^z \)

\[
\frac{\partial g}{\partial x} = 2x ye^z, \quad \frac{\partial g}{\partial y} = x^2 e^z, \quad \frac{\partial g}{\partial z} = x^2 ye^z
\]

15. If the consumption is $8 billion when disposable income is 0, and if the marginal propensity to save is \( \frac{dS}{dy} = 0.5 + e^{2.3y} \) (in billions of dollars), find the national consumption function.

\[
\frac{dS}{dy} = 0.5 + e^{2.3y} \implies \frac{dS}{dy} = 0 - 0.5ye^{2.3y}
\]

\[
C = \int \left( 0.5 - e^{2.3y} \right) dy = 0.5y - \frac{e^{2.3y}}{2.3} + k
\]

but \( C = 8 \text{ when } y = 0 \implies 8 = 0 - \frac{1}{2.3} + k \implies k = 8.43
\]

\[
\Rightarrow C(y) = 0.5y - \frac{1}{2.3} e^{2.3y} + 8.43
\]

16. Given \( f(x, y) = \ln(x^2 + 2y) + x^4 - 2y^3 + xy \) find the following partial derivatives.

(a) \( f_x = \frac{2x}{x^2 + 2y} + 4x^3 + y = 2x(x^2 + 2y)^{-1} + 4x^3 + y \)

(b) \( f_y = \frac{2}{x^2 + 2y} - 6y^2 + x = 2(x^2 + 2y)^{-1} - 6y^2 + x \)

(c) \( f_{xy} = -2x(x^2 + 2y)^{-2}(2y) - 1 = -4x(x^2 + 2y)^{-2} + 1 \)

(d) \( f_{xx} = 2(x^2 + 2y)^{-4} + 2x(-1)(x^2 + 2y)^{-2}(2x) + 12x^2 \)

(e) \( f_{yy} = -2(x^2 + 2y)^{-3}(2y) - 12y \)

17. For the function given by \( f(x, y, z) = 2xy z^2 + x^3 y^2 z - y^4 \) find the following partial derivatives.

(a) \( f_x = 2yz^2 + 3x^2 y^2 z \)

(b) \( f_{xx} = 4yx + 3x^2 y^2 \)

(c) \( f_{xyz} = f_{xzy} = 4yz + 6x^2 y \)

(d) \( f_y \) when \( x = 1, y = 0 \) and \( z = 2 \)

\[
f_y = 2x^2 + 2xy z - 4y^3
\]

\[
f_y(1, 0, 2) = 2(1)(2) + 2(1)(0)(2) - 4(0) = 8
\]
18. Suppose that a product has marginal revenue given by $MR = 75$ and marginal cost given by $MC = 40 + \frac{5}{2}x$. If the fixed cost is $105$, how many units will give the maximum profit and what is the maximum profit?

\[ MC(x) = 40 + 2.5x \Rightarrow C(x) = 40x + 1.25x^2 + D \]
\[ MC(0) = 105 \Rightarrow D = 105 \Rightarrow C(x) = 40x + 1.25x^2 + 105 \]

\[ MR(x) = 75 \Rightarrow R(x) = 75x + K \quad \text{but} \quad R(0) = 0 \Rightarrow K = 0 \Rightarrow R(x) = 75x \]

\[ p(x) = 75x - (40x + 1.25x^2 + 105) = -1.25x^2 + 35x - 105 \]

\[ p'(x) = -2.5x + 35 = 0 \quad \Rightarrow \max \text{ profit at } x = 14 \]
\[ p(14) = 140 \quad \max \text{ profit} \]

19. Given the function $f(x, y, z) = \frac{2x^2 + \ln z}{\sqrt{2y + 6}}$

(a) Evaluate $f(1, 5, 1)$.

\[ f(1, 5, 1) = \frac{2(1) + \ln 1}{\sqrt{10 + 6}} = \frac{2 + 0}{4} = \frac{1}{2} \]

(b) Find the domain of $f(x, y, z)$.

\[ x > 0, \quad 2y + 6 > 0 \quad (\Rightarrow \quad y > -3) \]

20. A certain firm’s marginal cost for a product is $MC = 5x + 100$ and its marginal revenue is $MR = 180 - 2x$. The total profit of the production of 100 items is $15,000$.

(a) Find the total profit function.

\[ MP = MR - MC = 180 - 2x - 5x - 100 = 80 - 7x \]
\[ \Rightarrow p(x) = \int (80 - 7x) \, dx = 80x - 3.5x^2 + D \]
\[ p(100) = 15000 \Rightarrow 15000 = 80(100) - 3.5(100)^2 + D \]
\[ \Rightarrow D = -12000 \]

(b) Determine the level of production that yields the maximum profit.

\[ p'(x) = 80 - 7x = 0 \]
\[ x = \frac{80}{7} \approx 11.43 \]

\[ x = 11.43 \quad \text{produces} \quad \max \text{ profit} \]
21. If $1000 is invested for $x$ years at 8% compounded continuously, the future value of the investment is given by $S(x) = 1000e^{0.08x}$.

(a) Find the function that gives the rate of change of this investment.

$$
\frac{dS}{dx} = S' = 1000e^{0.08x} \\
(0, 80) = 80e^{0.08} 
$$

(b) Compare the rate at which the future value is growing after 1 year and after 10 years.

$$
S'(1) = 80e^{0.08} \approx 86.66 \\
S'(10) = 80e^{0.8} \approx 178.04
$$

22. The marginal cost for a product is $MC = 12x + 20$ dollars per unit, and the cost of producing 50 items is $1,300. Find the total cost function.

$$
C(x) = \int (12x + 20) \, dx = 6x^2 + 20x + C \\
C(50) = 1300 \Rightarrow 6(50^2) + 20(50) + C = 1300 \Rightarrow C = -14700 \\
C(x) = 6x^2 + 20x - 14700
$$

23. If the graph below represents the graph of $y = f(x)$, answer the following questions.

(a) $\lim_{x \to 6} f(x) = \infty \text{ or DNE}$

(b) $\lim_{x \to 5^-} f(x) = 6$

(c) $\lim_{x \to 5^+} f(x) = 4$

(d) $\lim_{x \to 5} f(x) \text{ DNE}$

(e) $\lim_{x \to 13} f(x) = 2$

(f) $\lim_{x \to 18} f(x) = 5$

(g) $f(-6) \text{ DNE}$

(h) $f(5) = 4$

(i) $f(13) = 3$

(j) $f(18) \text{ DNE}$

(k) For what x-values is $y = f(x)$ discontinuous?

$\cup \quad \text{VA} \quad \text{jump} \quad \text{hole}$

@ $x = -6, 5, 13$
24. Suppose a continuous income stream has an annual rate of flow \( f(t) = 85e^{-0.01t} \), in thousands of dollars per year, and the current interest rate is 7% compounded continuously.

(a) Find the total income over the next 12 years.

\[
I = \int_0^{12} 85e^{-0.01t} \, dt = \left. -\frac{8500e^{-0.01t}}{-0.01} \right|_0^{12} = 8500(e^{-0.12} - e^0) = 9411.18
\]

(b) Find the present value over the next 12 years.

\[
PV = \int_0^{12} 85e^{-0.01t} e^{-0.07t} \, dt = 85 \int_0^{12} e^{-0.08t} \, dt = \left. -\frac{8500e^{-0.08t}}{-0.08} \right|_0^{12} = 8500(e^{-0.08(12)} - e^0) = 8555.68
\]

(c) Find the future value 12 years from now.

\[
FV = e^{0.07(12)} \left( 8500(e^{-0.08(12)} - e^0) \right) = e^{0.09(12)} \left[ 8555.68 \right] \\
\approx 15187.9
\]

25. Suppose the supply function for a product is \( f(x) = 40 + 0.001x^2 \) and the demand function is \( p = 120 - 0.2x \), where \( x \) is the number of units and \( p \) is the price in dollars. If the market equilibrium price is $80, find the following.

(a) the consumer's surplus

\[
CS = \int_0^{x_1} f(x) \, dx - p \cdot x_1 = \int_0^{200} (120 - 0.2x) \, dx - 80(200) = \left. (120x - 0.1x^2) \right|_0^{200} - 16000 = 4000
\]

(b) the producer's surplus

\[
PS = p \cdot x_1 - \int_0^{x_1} g(x) \, dx = 80(200) - \int_0^{200} (40 + 0.001x^2) \, dx = 16000 - \left. \left( 40x + \frac{0.001}{3}x^3 \right) \right|_0^{200} = 16000 - (40(200) + \frac{0.001(200)^3}{3} - 0) \\
\approx 5333.33
\]
26. The cost of producing $x$ cupcakes is given by $C(x) = 100 + 20x + 0.01x^2$ dollars. How many units should be produced to minimize average cost?

$$\bar{C} = \text{avg cost} = \frac{C(x)}{x} = \frac{100}{x} + 20 + 0.01x$$

$$\bar{C}'(x) = -\frac{100}{x^2} + 0.01 = 0$$

$$\Rightarrow \frac{1}{100} = \frac{1}{x^2} \Rightarrow x^2 = 100 \Rightarrow x = 100 \text{ (since x can't be negative)}$$

27. The demand function for a product under competition is $p = \sqrt{64 - 4x}$ and the supply function is $p = x - 1$, where $x$ is the number of units and $p$ is in dollars. Find the following.

(a) the market equilibrium point

$$\sqrt{64-4x} = x-1$$

$$64-4x = (x-1)^2$$

$$64-4x = x^2 - 2x + 1$$

$$x^2 + 2x - 63 = 0$$

$$x = 7, -9$$

Equilibrium pt at $x = 7$

$$p = x - 1 = 7 - 1 = 6$$

$(7, 6)$ equil. pt

(b) the consumer's surplus at market equilibrium

$$CS = \int_0^7 (\sqrt{64-4x}) - x \, dx = \int_0^7 64-4x - x \, dx - 6(7)$$

$$= \int_0^7 u \frac{3}{2} (\frac{2}{3}) du - 42 = \frac{1}{4} u^3 \bigg|_{x=0}^{x=7} - 42$$

$$= \frac{1}{6} (64 - 4x)^{3/2} \bigg|_0^7 - 42$$

$$= \frac{1}{6} (216 - 512) - 42$$

$$= -\frac{1}{6} (296) - 42$$

$$= -$1.33

(c) the producer's surplus at market equilibrium

$$PS = p(x_1) - \int_0^{x_1} g(x) \, dx$$

$$= 6(7) - \int_0^7 (x-1) \, dx = 42 - \left(\frac{x^2}{2} - x\right) \bigg|_0^7$$

$$= 42 - \left(\frac{49}{2} - 7\right) = $24.50