3.1 Quadratic Equations in One Variable

Ex 1 Solve these equations.

(a) $5x^2 - 32 = x^2 + 8$

(b) $x^2 - 24 = -5x$

Quadratic Eqn has form:

$ax^2 + bx + c = 0$

$a, b, c \in \mathbb{R}$, $a \neq 0$

Techniques to Solve

① **Square Root Technique:** only works if you can get your variable to appear in the eqn <u>one</u> time.

② **Factoring:** only works if quadratic expression factors.

③ **Completing The Square:** always works.

note: you must factor out coefficient of x^2 first before completing the square

④ **Quadratic Formula:** always works.

Given $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

IMPORTANT: make sure you have everything on one side w/ zero on other side of = sign.

3.1 (cont)

Ex 2 Solve these equations.

(a) $x^2 - 3x + 1 = 0$

(b) $3x^2 + 12x - 4 = 0$

(c) $2x^2 + 6x = 5$

(d) $\frac{2x}{x-2} - \frac{x}{x+3} = 2$

3.2 Parabolas: Quadratic Equations in Two Variables

Ex 1 Find the (a) vertex, (b) tell whether it's max or min, (c) decide if parabola is concave up or down, and (d) find axis of symmetry.

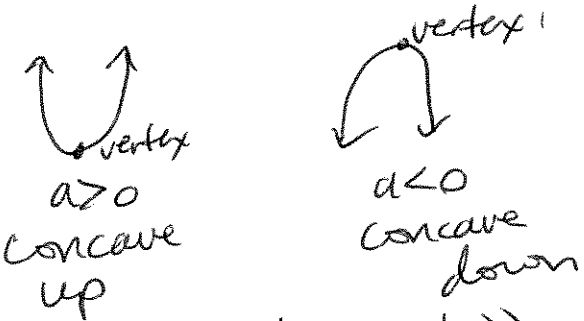
(a) $y = x^2 - 4x + 3$

(b) $y = -5x^2 + 10x - 1$

Quadratic Eqn in 2 Variables

$f(x) = ax^2 + bx + c$

$a, b, c \in \mathbb{R}$, $a \neq 0$



vertex at: $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

other form of parabola

$f(x) = a(x-h)^2 + k$

(h,k) vertex

axis of symmetry: $x = h$

roots/zeros of quadratic fn are x-values where $y = 0$.

these then give you the x-intercepts

3.2 (cont)

Ex 2 Find (i) vertex, (ii) axis of symmetry, (iii) concavity, & (iv) x-intercepts. Then sketch the parabola.

(a) $y = -(x-3)^2 + 4$

(b) $y = 2x^2 + 4x + 2$

Ex 3 A rectangular playground is to be fenced in on all four sides. There is exactly 600 ft. of fencing for this playground. What dimensions give the largest area for the playground?

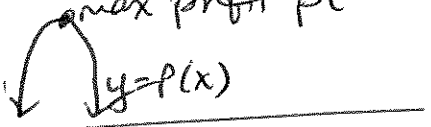
3.3 Quadratic Business Applications

Ex 1 Find max profit if the revenue per watch sold is $10. and cost is $C(x) = (x+1)^2 - 499$.

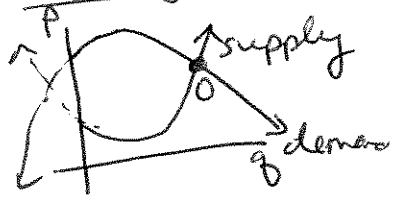
Ex 2 If demand is $p = 2000 - 6q$, find max revenue. (Hint: Revenue = # of units sold times price.)

① Profit/Revenue/Cost

- $P = R - C$
- C = fixed costs + variable costs
- break-even pts occur where $R = C$ (or $P = 0$)



② Supply/Demand



O is at equilibrium pt where supply & demand match

q = quantity
p = price

Ex3 If demand for a service is given by $p = -q^2 - 4q + 64$ and supply is given by $p = q^2 + 8q + 10$, find the equilibrium pt.

Ex4 A company has production costs of $C(x) = 800 - 10x + 0.25x^2$. How many items should be produced to minimize costs?

3.4 Polynomial Functions

Ex 1 For each polynomial,
(i) rewrite in descending order.
(ii) find its degree.
(iii) what is leading coefficient?
(iv) draw general shape.

(a) $f(x) = 2x + x^6 - 3x^5 + 4$

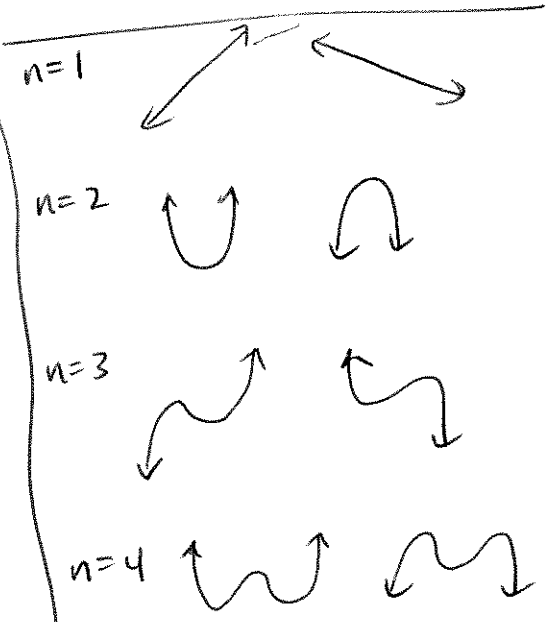
(b) $f(x) = 1 - 4x^3 + x^2$

polynomial fn:

$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$

a_i are constants

ex $f(x) = x^3 + 5x + 7$



degree = highest exponent on x

leading coefficient = constant multiplied by highest degree term

3.4 (cont)

Ex2 For each polynomial function, find the following:
(i) degree, (ii) zeros (roots), (iii) y-intercept, (iv) x-intercepts,
(v) sketch the graph.

(a) $g(x) = x(x+3)(x-2)$

(b) $f(x) = 2(x-1)^4$

Ex3 For each piecewise function, fill in the table and sketch the graph

(a) $f(x) = \begin{cases} x+3 & x \geq 1 \\ -2x & x < 1 \end{cases}$

x	y
-1	
0	
1	
2	
3	

(b) $f(x) = \begin{cases} 2x-1 & x \geq 0 \\ x+5 & -3 \leq x < 0 \\ -x & x < -3 \end{cases}$

x	y
-4	
-3	
-1	
0	
1	
2	

3.5 Rational Functions

Ex 1 Find VA and HA for these rational fns.

(a) $f(x) = \frac{3x}{x-2}$

(b) $f(x) = \frac{3x-15}{(x-1)(x+2)}$

(c) $f(x) = \frac{x^3+x+7}{x(x+3)}$

rational fn: a fn that's a fraction w/ polynomial in numerator and denominator.

$f(x) = \frac{n(x)}{d(x)}$

VA: occur where $d(x) = 0$ (and $n(x) \neq 0$).

HA:

① If degree $n(x)$ > degree $d(x)$, there's no HA.

② If degree $n(x)$ = degree $d(x)$, then $y = \frac{\text{leading coeff } n(x)}{\text{leading coeff } d(x)}$

③ If degree $n(x)$ < degree $d(x)$, HA is $y = 0$.

3.5 (cont)

EX2 For each rational fn, find

(i) domain, (ii) VA, (iii) HA, (iv) y-intercept,
(v) x-intercept(s), (vi) sketch graph.

(a) $f(x) = \dfrac{(x-1)(x+2)}{x^2+1}$

(b) $f(x) = \dfrac{x(x-2)(x+5)}{(x-5)(x+1)}$

3.6 Transformations of Graphs

Ex 1 For each fn, answer these questions.

(a) what is vertical shift?
(b) " " horizontal shift?
(c) Is graph stretched/shrunk horizontally? If so, how?
(d) Is graph stretched/shrunk vertically? If so, how?
(e) Is graph reflected vertically?
(f) " " " horizontally?

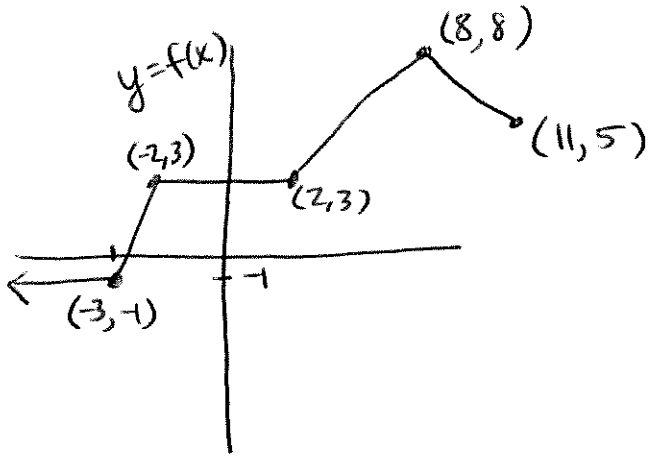
(a) $f(x) = -3(x-1)^2 + 9$

(b) $g(x) = -\sqrt{2x} + 5$

For Base Graph $y = f(x)$:

$h(x) = f(x) + c$	shifts up c (if $c > 0$) or shifts down $\lvert c \rvert$ (if $c < 0$)
$h(x) = f(x+c)$	shifts left c ($c > 0$) or shifts right c (if $c < 0$)
$h(x) = -f(x)$	reflects vertically
$h(x) = f(-x)$	reflects horizontally
$h(x) = c\,f(x)$	stretches/shrinks graph vertically (stretch if $c > 1$, shrink if $0 < c < 1$)
$h(x) = f(cx)$	stretches/shrinks graph horizontally (stretch if $0 < c < 1$, shrink if $c > 1$)

EX2 Given this graph (base graph), sketch the following.



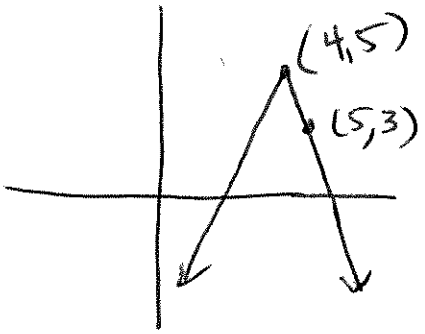
(a) $f(-x)+1=y$

(b) $y=-f(x+3)$

(c) $y=-f(-x)$

3.6 (cont)

Ex3 Write an function that would produce this graph. The base function is _____.



Ex4 Given $f(x) = -3(x+1)^2 + 2$, fill in the table and then sketch the graph.

base $f(x)=x^2$	$y=-x^2$	$y=-3x^2$	$y=-3(x+1)^2$	$y=-3(x+1)^2+2$
(0,0)				
(1,1)				

3.7 Combinations of Functions

Ex 1 Given $f(x) = \frac{2}{x}$ and $g(x) = \sqrt[3]{x^2+1}$, find

(a) $(fg)(x)$

(b) $(f-g)(x)$

(c) $(f+g)(1)$

(d) $f(g(x))$

Addition

$(f+g)(x) = f(x) + g(x)$

Subtraction

$(f-g)(x) = f(x) - g(x)$

Multiplication

$(f \cdot g)(x) = f(x)\,g(x)$

Division

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

Composition

$(f \circ g)(x) = f(g(x))$

Note: $f(g(x)) \neq g(f(x))$ in general

Ex 2 Given $f(x) = 3x+1$, $g(x) = x^3$, $h(x) = \frac{1}{\sqrt{x}}$

find the following

(a) $\left(\frac{g}{h}\right)(x)$

(b) $(g \circ f)(x)$

(c) $(f \circ f)(x)$

(d) $f(g(h(1)))$

Ex 3 Find an $f(x)$ and $g(x)$ such that $(f \circ g)(x) = (x-1)^3 + 2$.
(there are many answers)