

Math5700 Notes
Section 5.1
Induction and Recursion

Recursion formulas have two components:

- (1) Initial values
- (2) Recurrence relation (formula relating the next term in the sequence to one or more previous terms)

We can guess at an iterative (direct) formula, given the recursion definition, and then prove our claim using proof by induction.

Note: We need strong induction if the recursive formula depends on more than one previous term. But, we can use weak induction if the recursive formula depends only on one previous term.

Steps for Proof by Induction:

Ex 1: Use the recursive definition of x^m and induction to prove that $\forall m, n \in \mathbb{N} (x^m)^n = x^{mn}$.

Ex 2: Given the relationship $a_0=0$ and $a_n=a_{n-1}+n^2$, conjecture a direct formula for a_n and then prove it.

Ex 3: Conjecture a simple formula for $p(n) = \left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right) \dots \left(1 - \frac{1}{n^2}\right)$, $n \in \mathbf{N}$, and then prove it.