Math5700 Notes Section 3.3.3 Extended Analysis of some FUN problems

Starter:

Dirt Biker Problem

A dirt biker must circle a 5-mile track twice. His average speed must be 60 mph. On his first lap, he averaged 30 mph. How fast must he travel on his second lap in order to qualify?

Generalize your result, assuming the track is d miles around and the speed on the first lap is r mph.

Extended Analysis of the Box Problem

Find the dimensions of an open box with maximum volume that can be made from a 12×15 inch rectangular sheet of cardboard by cutting small squares out of the corners, folding up the sides and taping the corners.

1. Can algebra students do this? If so, how?

2. Use calculus to solve the problem exactly. Let x be the length of the square corners that are cut out of the original cardboard piece.

3. Let's extend the problem. Let the width of the rectangular cardboard sheet be fixed at w = 1 and let the length, L, be variable. Use calculus to solve the problem again.

4. If L = 1, what is x?

5. If L is huge, then x is very small and L-2x is almost L. This means that maximizing volume here is almost the same as maximizing A = x(1-2x)...what is this equivalent to?

And if L is huge, what does the max volume approach? Go back to #2. What is the "base area" of the box?

What is the "wall area" of the box?

Let's Generalize some more!! :)

6. What if we have a regular hexagon cardboard piece, rather than a rectangle? Solve the problem with this shape, assuming we still want to cut off the corner pieces to fold them up and create an open-topped box.

Let L = the length of each leg of the regular hexagon cardboard piece.

What is the "base area" and "wall area" for this box?

7. Can we solve the "same" problem again if we start with an equilateral triangle piece of cardboard?