Math5700 Notes
Mystery Notes

(For this lecture, you'll have to suspend some mathematical ideas that you already think you know, so that we can explore this as if for the first time!)

$$D_x(\frac{x^3}{3}) = x^2$$
$$D_x(\frac{x^2}{2}) = x$$
$$D_x(\frac{x}{1}) = 1$$
$$D_x(\ ??\ ) = x^{-1}$$
$$D_x(\frac{-1}{x}) = x^{-2}$$
$$D_x(\frac{-1}{2x^2}) = x^{-3}$$

What function goes in the place of the ??? This is our driving question. In other words, we are on the hunt for the function whose derivative is $\frac{1}{x}$.

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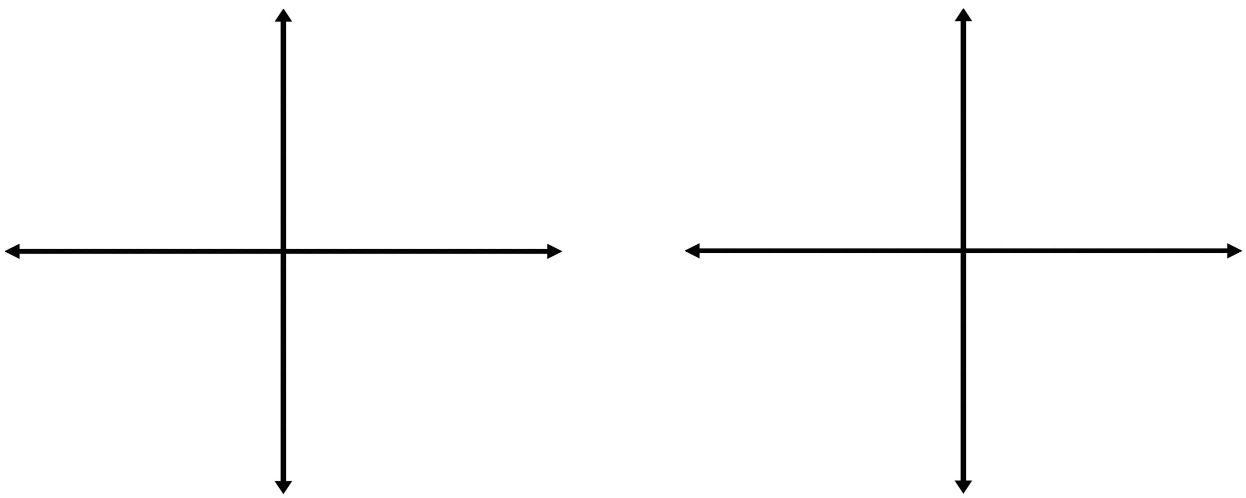
Remember what the First Fundamental Theorem of Calculus states?

For now, we need to create our own symbol that represents this function that we know exists for all $x > 0$.



Our new function is well defined for $x > 0$, but not defined for $x < 0$ (nor for $x = 0$) because we can't integrate over the interval containing $x = 0$.

$\Longrightarrow$

Claim:

Proof:

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This fills in our gap, namely ___________________________________.

Properties of our new operator:
$a, b \in \mathbb{R}^+$, $r \in \mathbb{Q}$

(i)

(ii)

(iii)

(iv)

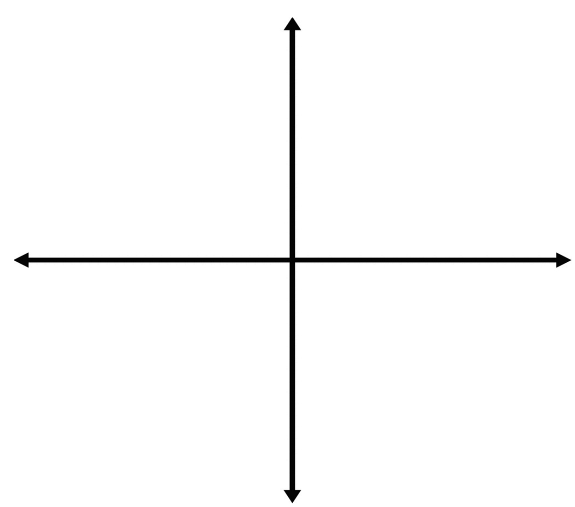
Proof:
(i)

(ii)

(iii)

(iv)

Let's investigate the shape of our new function.



We can see that an inverse exists!

Let's define the inverse by some new function (that also needs new notation/symbol) as:

Definition:

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Our definition of e is that it's the constant such that ln(e) = 1.

Let's look at e two different ways, in addition to our definition.

(1) Prove that $e = \lim_{h \to 0} (1+h)^{\frac{1}{h}}$ (or equivalently $e = \lim_{n \to \infty} (1+\frac{1}{n})^{n}$) .

(2) Prove that $e=\sum_{n=0}^{\infty} \frac{1}{n!}$.