

Compass & Straightedge Properties

- (1) For all $r > 0$ and a point C , we can construct a circle of radius r and with center C . (An arc is considered to be a connected portion of a circle.)
- (2) Every two points P & Q can be connected using our straightedge to construct the line segment, ray or line between them.
- (3) A line l can be constructed if and only if we have two points that are on line l . The points are located only by the intersection of lines, segments, rays or arcs.

Basic Constructions

- (1) Copy a line segment →
 - Draw another segment (that looks longer than the one you're copying).
 - Open the compass to the length of the original segment (by putting compass point on one endpoint and compass pencil on other endpoint).
 - Put the compass point on the line segment you drew (at some point you've designated), draw an arc (with the compass setting from last step) that intersects that line segment. The line segment between those two points is now your copied segment.
- (2) Copy an angle (call it angle A) →
 - Draw another line segment, label one endpoint as P .
 - Draw an arc through angle A (with compass point on vertex A).
 - With the same compass setting as last step, put compass point on P and draw an arc (making sure it goes through the line segment you've drawn and call that intersection point R).
 - With compass, measure distance from arc intersection points with angle A .
 - With that same compass setting, place the compass point at R and draw another arc. Call the point of intersection, with this arc and the last arc around P , point Q .
 - Connect (with your straightedge) point Q and point P and you now have your copied angle.

- (3) Bisect (perpendicularly) a line segment (call endpoints A and B) →
- Place compass at point A and open the compass to a setting that looks bigger than half the distance between A and B (but less than the whole distance). Construct arcs on both sides of line segment.
 - With the same compass setting, place the compass at B and make arcs that intersect those arcs you've already drawn. Call those two new points P and Q.
 - Connect P and Q with your straightedge. That new line segment is a perpendicular bisector of your original line segment.
- (4) Bisect an angle (call it angle B) →
- Place compass point at vertex B and draw an arc that goes through both legs of the angle. Label those intersection points P and Q.
 - Place compass point at P and draw an arc (you can choose your compass setting).
 - With the same compass setting as in last step, place your compass point at Q and draw an arc. Label the point of intersection between the two arcs as point R.
 - With your straightedge, connect B and R. That line segment bisects the angle.
- (5) Construct a line perpendicular to a given line through a specified point on the line (call that point P) →
- Place the compass point at P and draw two arcs that intersect the line on either side of P (each with same compass setting). Label those two points S and R.
 - Bisect the straight angle formed by those three points (namely P, S, and R) by above construction process.
 - The constructed line segment that bisects the straight angle is perpendicular to the original line and goes through P.
- (6) Construct a line perpendicular to a given line through a specified point not on the line (call that specified point P) →
- Place the compass point at P and draw two arcs that intersect line on either side of P (both with same compass setting). Label those two points (on the line) as A and B.
 - With the same compass setting as in last step, place compass point at A and draw an arc on the other side of the line from P.
 - Still with the same compass setting, place compass point at B and draw another arc on the other side of the line from P so that this arc intersects arc from last step. Label that point Q.
 - With your straightedge, draw the line segment that connects the points P and Q. This line segment is perpendicular to the original line and goes through P.

- (7) Construct a line parallel to a given line through a given point (call that point P) →
- Designate some point on the line as Q. With your straightedge, create a line segment connecting P and Q.
 - Designate another point on the original line as R. Now use the technique of copying an angle to copy angle PQR to angle having its vertex at P (that will form an alternate interior angle with angle PQR).
 - When you've copied that angle, the new line segment created will be parallel to the original line.

More Euclidean Constructions

- (8) Construct circumscribed circle of a triangle (call triangle vertices A, B & C) →
- Construct a perpendicular bisector line of line segment AC.
 - Construct a perpendicular bisector line of line segment AB.
 - Label the intersection of those two created lines as point P.
 - Draw the circle with center P (called *circumcenter*) and radius as length from P to A (or B or C). This circle should go through all vertices of original triangle.
- (9) Construct inscribed circle of a triangle (call triangle vertices A, B & C) →
- Construct the angle bisector of angle A.
 - Construct the angle bisector of angle B.
 - Label the intersection of those two created lines (the angle bisector lines) as point P.
 - Construct a line through P and perpendicular to line segment AB. Label the intersection point as R.
 - Draw the circle with center P (called *incenter*) and radius as length of line segment PR. This circle should just touch all three sides of the original triangle.
- (10) Construct an equilateral triangle (call it triangle ABC) →
- Choose points A and B at random and draw the line segment connecting them using your straightedge.
 - Place the compass point at A and measure length to B.
 - With that compass setting, draw an arc from A.
 - With the same compass setting, draw an arc from B.
 - Label intersection point, from two arcs, as point C.
 - With your straightedge, make line segments connecting A to C and then B to C. You've created an equilateral triangle through points A, B and C.