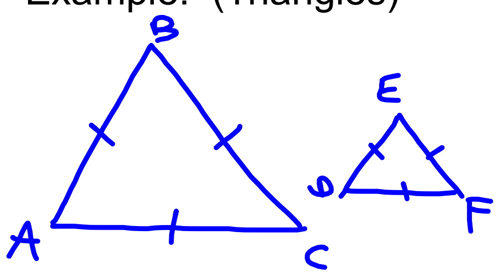
12.1-12.2 Congruence Properties and Constructions

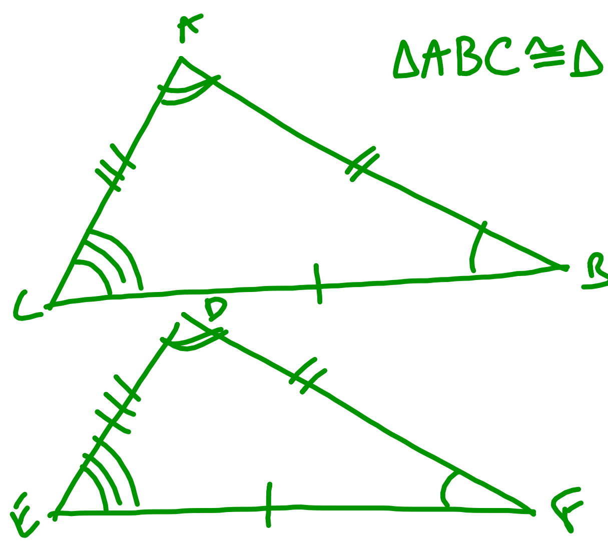
Congruent-- two shapes are congruent if every corresponding measurable quantity for each of them is the same. (Notation: ≅)

Similar--two shapes are similar if they are the exact same shape, but they are scaled versions of each other. (Notation: ~)

Example: (Triangles)



$\Delta ABC \sim \Delta DEF$



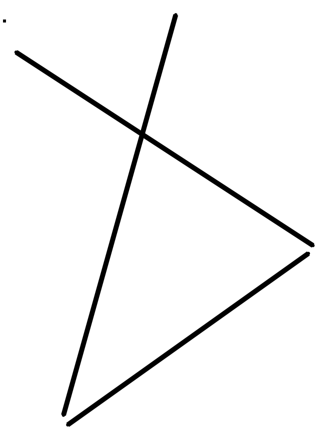
$\Delta ABC \cong \Delta DFE$

1 Draw a segment of length 6 cm on your paper.
Do not make it horizontal or vertical. (ASA)

At one end, draw a 40º angle.

At the other end draw a 70º angle.

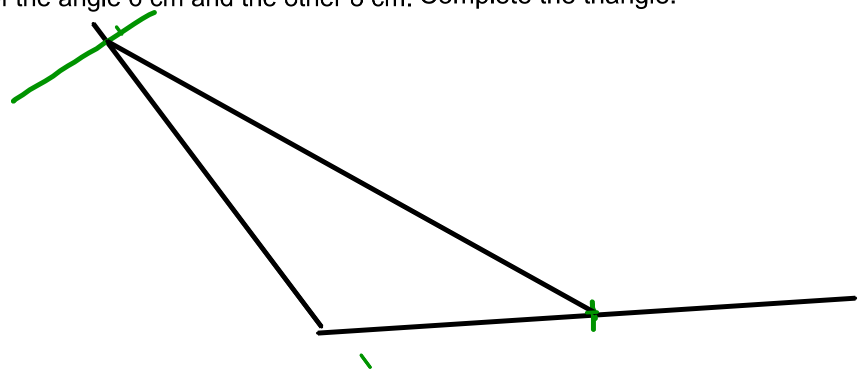
Complete the triangle. Label it with your initials. Compare it to your partners' triangles.



②

Draw a 120º angle on your paper. (SAS)

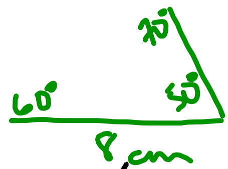
Make one side of the angle 6 cm and the other 8 cm. Complete the triangle.

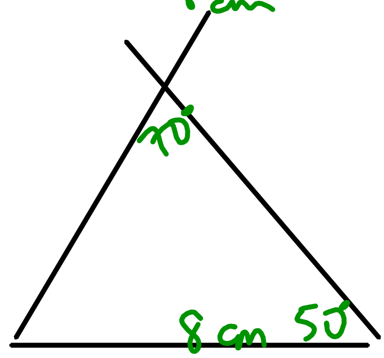


③ (AAS)

Draw a triangle with these parts - in order

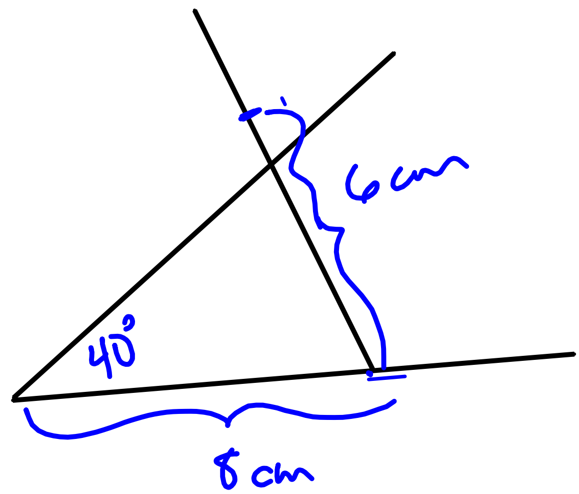
Side 8 cm. Angle 50º Next angle (not on the 8cm. segment) 70º





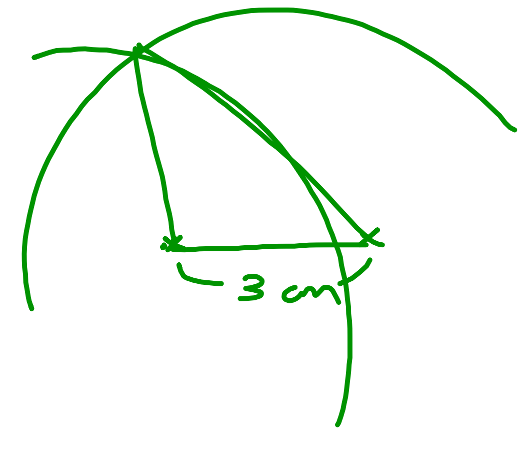
④ Draw this one: (SSA)

Angle = 40º Side = 8 cm Side = 6cm
The angle is NOT between the two sides.



5 Draw a triangle with these three sides:
3 cm 5 cm 6 cm

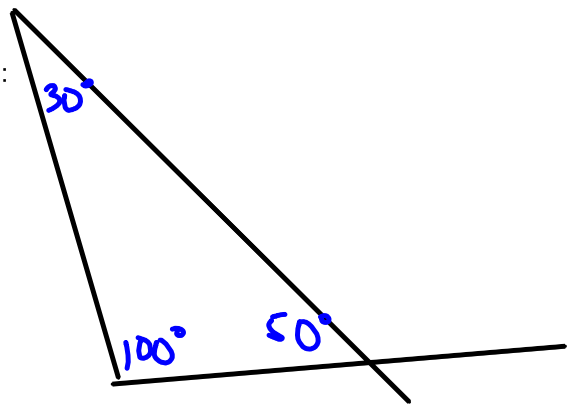
(SSS)



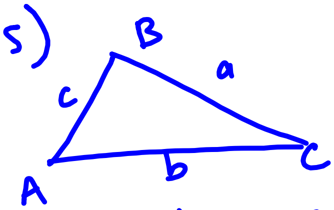
⑥

Draw one with these three angles:

50º, 30º, 100º



Congruence Theorems: (for triangles)



six measurable quantities

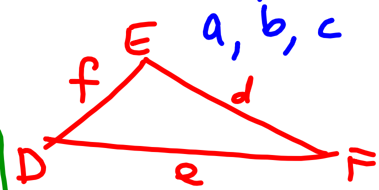
$m\angle A$, $m\angle B$, $m\angle C$

a, b, c

SSS : $c=f$, $a=d$, $b=e$
(5)
$\Rightarrow \triangle ABC \cong \triangle DEF$

SAS (side, angle, side)
(2)
↳ between the two side
ex $c=f$, $a=d$, $B=E$

ASA (angle, side, angle)
(1)
↳ between 2 angles
ex $A=D$, $C=F$, $b=e$



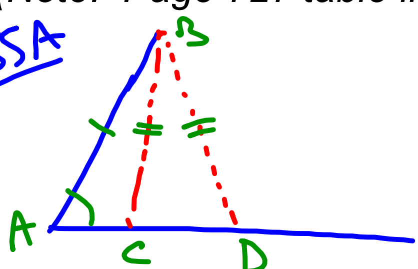
AAS (angle, angle, side)
(3)
↳ not bet. 2 angles

SSA (side, side, angle).
(4)
↳ not included bet. sides
ambiguous

(Note: Page 727 table in book is really useful.)

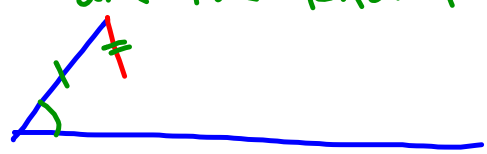
SSA

(green marks means these are the known quantities)

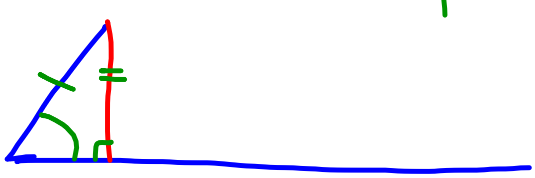


case 1

2 possible $\triangle$s
$\triangle ABC$ or $\triangle ABD$

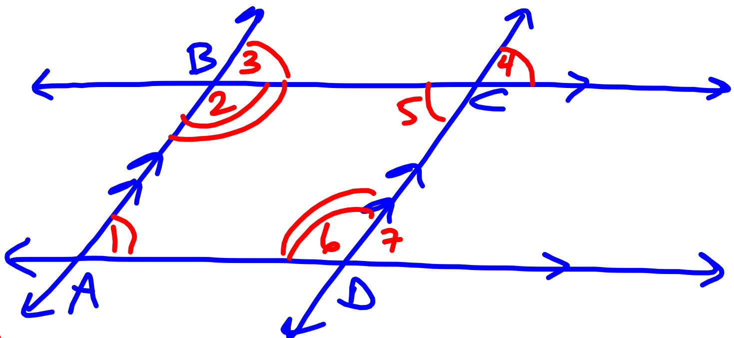


case 2
no possible $\triangle$s



case 3
1 possible $\triangle$

Prove that opposite sides and angles of a parallelogram are congruent.



$\square ABCD$ is parallelogram

Pf $m\angle 1 = m\angle 3$

① corresponding angles

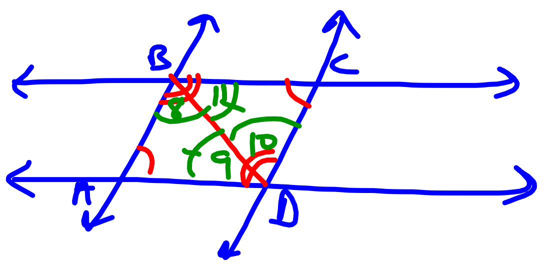
$m\angle 3 = m\angle 4$

$m\angle 1 = m\angle 4$ (transitivity)

$m\angle 5 = m\angle 4$ (vert. angles)

$m\angle 1 = m\angle 5$ (transitivity)

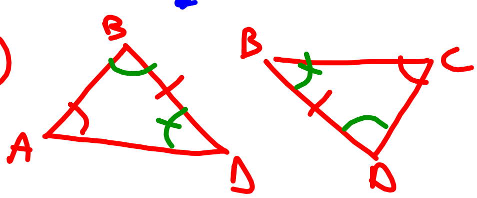
② By same logic as in ①, $m\angle 2 = m\angle 6$.



$m\angle 11 = m\angle 9$ alt. int. angles

$m\angle 8 = m\angle 10$ " "

③



by ASA, $\triangle ABD \cong \triangle CDB \Rightarrow \overline{AB} \cong \overline{CD}$

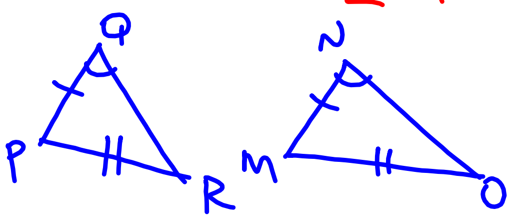
$\overline{BC} \cong \overline{DA}$

HW

12.1B #5) (c) $\overline{PQ} \cong \overline{MN}$ ($\triangle PQR \cong \triangle MNO$?)

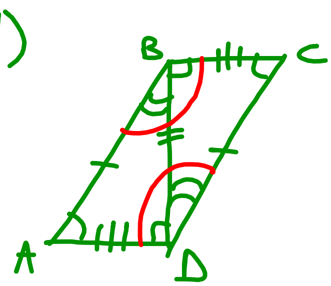
$\overline{PR} \cong \overline{MO}$

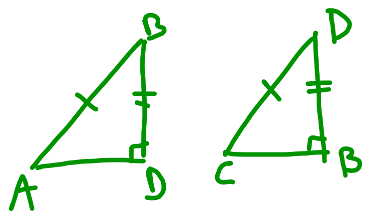
$\angle Q \cong \angle N$ NO



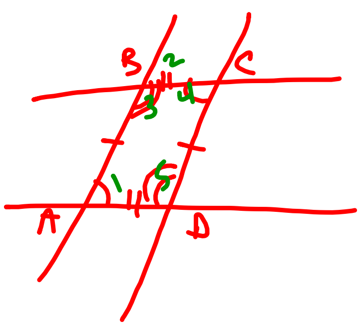
(SSA)

4)





by case 3 of SSA, $\triangle ABD \cong \triangle CDB$



claim $m\angle 1 = m\angle 2$

pf ① $m\angle 2 + m\angle 3 = 180°$

② $m\angle 1 + m\angle 3 + m\angle 4 + m\angle 5 = 360°$

$2m\angle 1 + 2m\angle 3 = 360°$

(because $m\angle 1 = m\angle 4$ and $m\angle 3 = m\angle 5$)

2a $\Leftrightarrow$ $m\angle 1 + m\angle 3 = 180°$

1 and 2a $m\angle 3 = 180° - m\angle 2$

and $m\angle 3 = 180° - m\angle 1$

$\Rightarrow$ $180° - m\angle 2 = 180° - m\angle 1$

$-m\angle 2 = -m\angle 1$

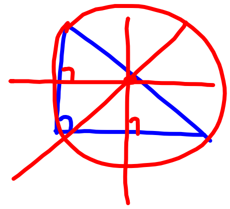
$m\angle 2 = m\angle 1$

So corresponding angles are $\cong$, $\Rightarrow \overline{AD} \parallel \overline{BC}$

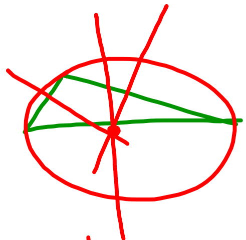
Similarly, $\overline{AB} \parallel \overline{CD}$

12.1 A #13)

(a)

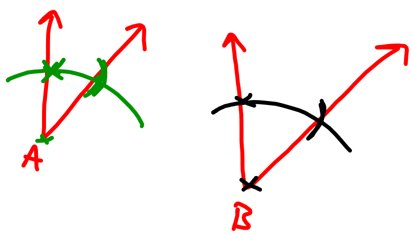


circumcenter: intersectn pt of $\perp$ bisectors of sides

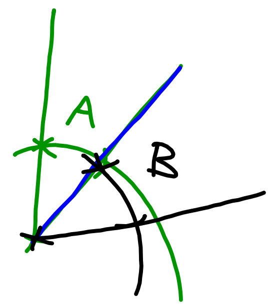


12.1A #7

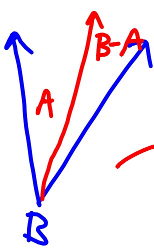
(a)



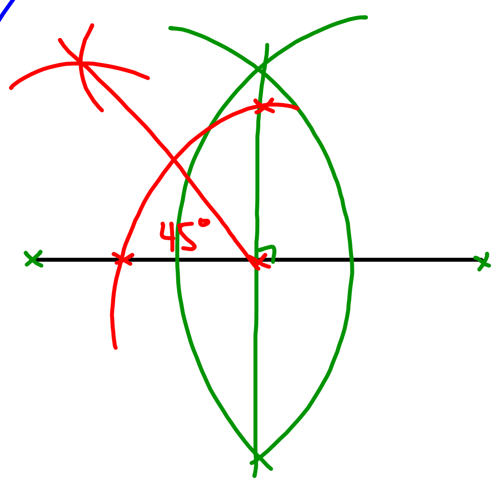
copy angle A



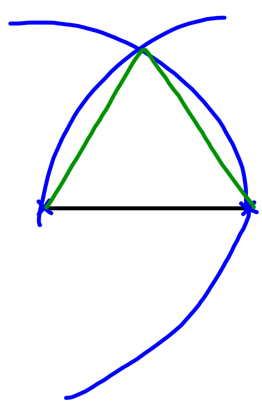
(b)



12.1 A#6



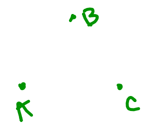
12.1B #6



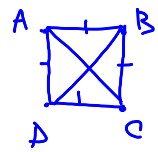
bisect 60° angle to get 30° angle

12.1B #2

3 pts equidistant from each other
(verts of equilateral Δ)

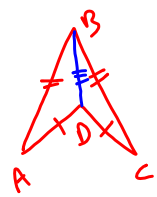


$AB=BC=AC$



$AB=BC=CD=AD \neq AC \neq BD$

12.2B #9

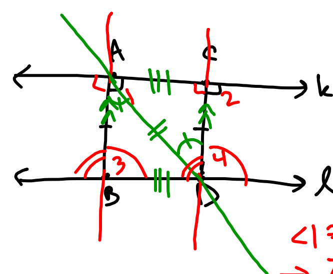


$\angle BAD \cong \angle BCD$?

by SSS, $\Delta ABD \cong \Delta CBD$

$\Rightarrow \angle BAD \cong \angle BCD$

12.2B #4



why is $k \parallel l$?

$\angle 1 \cong \angle 2$

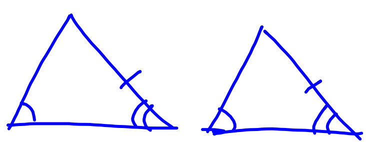
$\Rightarrow \overline{AB} \parallel \overline{CD}$

$\Rightarrow \angle 3 \cong \angle 4$ by corresponding angles

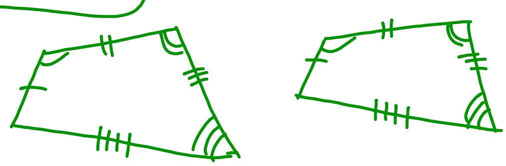
$\Rightarrow$ by SAS, $\Delta ACD \cong \Delta DBA$

lastly, argue $k \parallel l$ by alt. int. $\angle$s

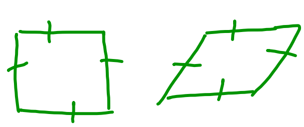
12.2MC #4



12.2 MC #6



7 pieces of info is enough for congruence



no