Chapter 17 | Inference about a Population Mean

☆ This chapter uses CI and significance tests for μ, assuming we do <u>NOT</u> know σ!
(which is more realistic)

<u>Assumptions</u>: ① We have an SRS.
② population is $N(\mu, \sigma)$ (both μ & σ unknown)
and (☆ population $\geq$ 20 * sample size)

<u>Standard Error</u>: standard deviation as calculated from sample data; standard error of $\bar{x}$ is $\frac{s}{\sqrt{n}}$.

<u>Ex 1</u> If $\bar{x} = 31.25$ for a sample size of 20, and $s = 21.88$, what is standard error of the mean?

<u>Ex 2</u> If a study reports "mean plus or minus standard error of the mean" as 251 ± 45, what are $\bar{x}$ and s? (For $n = 50$.)

(☆ Notice: this is not a CI.)
but some studies report this way

Chp 17 (cont)

One-sample t statistic and t distribution

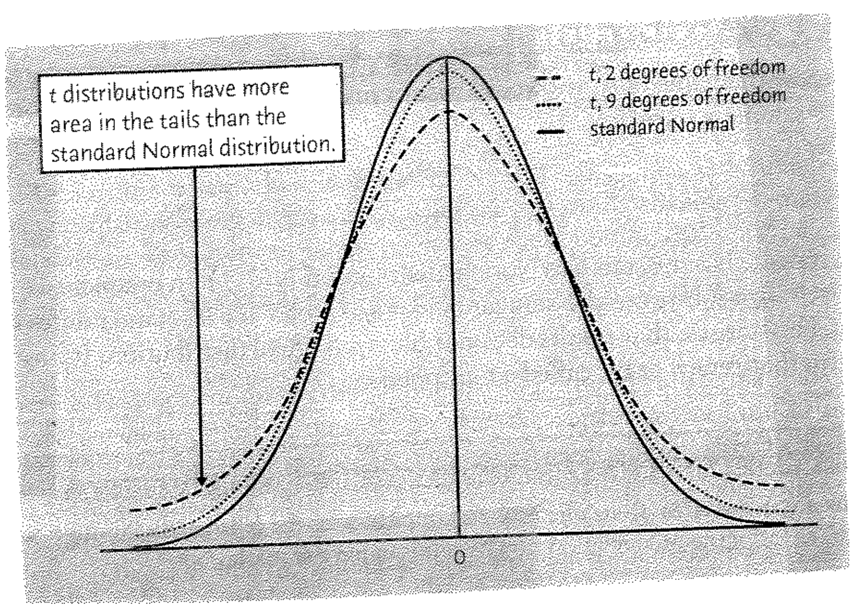
SRS of size n from large population $\sim N(\mu, \sigma)$.

one-sample t statistic : $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

has t distribution with n-1 degrees of freedom
notation: $t(n-1)$

☆ degrees of freedom come from s (remember from Chp 2, s has n-1 degrees of freedom)

- t-distribution curves similar to normal curves in shape
- t-distribution curves are more spread out than normal curves w/ more area under tails (+ less in center) (because using s instead of σ has more variation)
- as degrees of freedom increase t curve like N(0,1) (because as n increases, $s \simeq \sigma$)



Chp 17 (cont)

(Use Table C)

Ex 3 Find (a) critical value for one-sided test w/ $\alpha = 0.05$ based on $t(5)$ distribution

(b) critical value for a 98% CI based on $t(21)$ distribution.

Ex 4 You have SRS, $n = 25$, and you calculate one-sample t statistic. What is t^* (critical value) such that

(a) $P(t \geq t^*) = 0.025$

(b) $P(t \leq t^*) = 0.75$

Chp 17 (cont)

One-Sample t confidence interval (CI)

SRS size n from large population w/ μ = population mean.

A level C CI for μ is $\bar{x} \pm t^* \frac{s}{\sqrt{n}}$, t^* is critical value for $t(n-1)$ distribution with area C between $-t^*$ & t^*. (CI is exact if population normal & approximate for large n otherwise.)

Ex5 What is t^* to produce these CIs?

(a) 95% CI w/ $n=10$

(b) 99% CI w/ $n=20$

Ex6 Data from SRS: 63.4 65.0 64.4 63.3 54.8
64.5 60.8 49.1 51.0

Use 95% CI to estimate mean of population.

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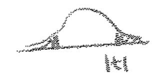
Chp 17 (cont)

One-sample t test

SRS of size n, from large population w/ mean μ. To test $H_0: \mu = \mu_0$, compute $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$ for $t(n-1)$ dist.

① $H_a: \mu > \mu_0$ p-value $= P(T \geq t)$

② $H_a: \mu < \mu_0$ p-value $= P(T \leq t)$



③ $H_a: \mu \neq \mu_0$ p-value $= 2P(T \geq |t|)$

Ex 7 $n = 25$ $H_0: \mu = 64$ one-sample t-statistic is
$H_a: \mu \neq 64$ $t = 1.12$

(a) degrees of freedom for t?

(b) t^*-values that contain t?
2-sided p-values for those t^* values?

(c) Is $t = 1.12$ statistically significant at 10% level?
At $\alpha = 0.05$ level?

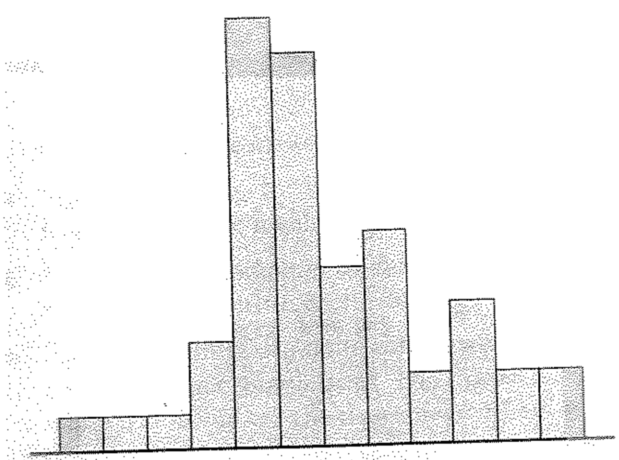
Chp 17 (cont)

Matched Pairs t procedure

To compare responses from 2 treatments in matched pairs design, find difference between responses w/in each pair. Then apply one-sample t procedures to that difference.

- SRS is more important than population being normal (except in case of very small sample size)
- $n < 15 \Rightarrow$ use t test if data looks normal; if data has outliers or is skewed, don't use t procedures
- $15 \le n < 40 \Rightarrow$ can use t procedures (except in cases of outliers or strong skewness)
- $n \ge 40$ (about) $\Rightarrow$ can use t procedures (even if skewed)

Ex 8 Can we use t-procedures on these data sets?

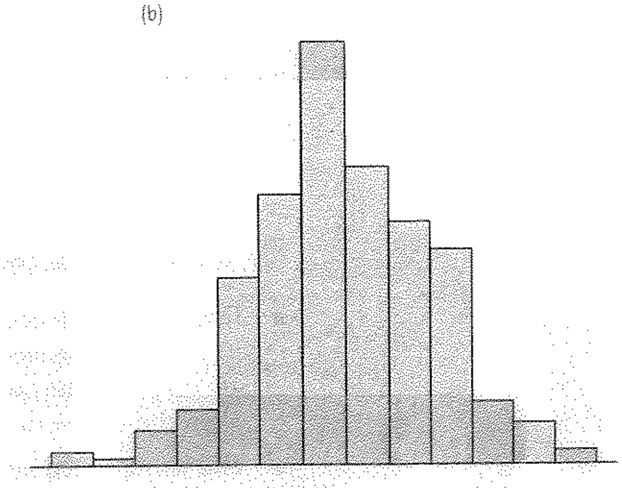

(a)

23	0
24	0
25	
26	5
27	
28	7
29	
30	2 5 9
31	3 9 9
32	0 3 3 6 7 7
33	0 2 3 6

(b)

37	4 8 9
38	0 0 1 1 2 2 8 9
39	2 6 8
40	6 7
41	5 7 9 9
42	0 2
43	1

(c)


(d)

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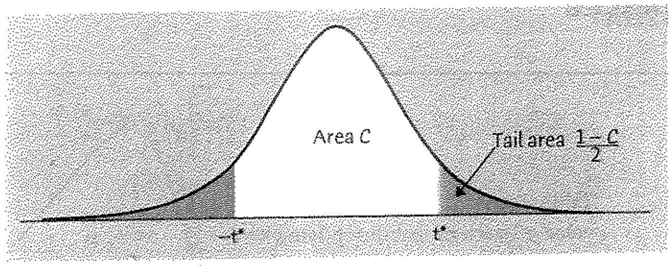
70

Table entry for C is the critical value t^* required for confidence level C. To approximate one- and two-sided P-values, compare the value of the t statistic with the critical values of t^* that match the P-values given at the bottom of the table.



BLE C *t* distribution critical values

REES OF EEDOM	CONFIDENCE LEVEL *C*											
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z^*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
ne-sided P	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
o-sided P	.50	.40	.30	.20	.10	.05	.04	.02	.01	.005	.002	.001