4.1 Mathematical Induction

Principle of Mathematical Induction

To prove $P(n)$ true $\forall n \in \mathbb{Z}^+$, where $P(n)$ is propositional fn:

1. Verify $P(1)$ true.
2. Show $P(k) \to P(k+1)$ true $\forall\ k \in \mathbb{Z}^+$.

$$\equiv \quad [P(1) \wedge \forall k\,(P(k) \to P(k+1))] \to \forall n\, P(n)$$

Ex 1 Prove $1\cdot 1! + 2\cdot 2! + \cdots + n\cdot n! = (n+1)! - 1$

4.1 (cont)

Ex 2 Prove $\sum_{j=0}^{n} \left(\frac{-1}{2}\right)^j = \frac{2^{n+1}+(-1)^n}{3\cdot 2^n}$.

Ex 3 Prove $2 - 2\cdot 7 + 2\cdot 7^2 - \cdots + 2(-7)^n = \frac{1-(-7)^{n+1}}{4}$ $\forall\, n \in \mathbb{W}$

4.1 (cont)

Ex 4 Prove $1+\frac{1}{4}+\frac{1}{9}+\cdots+\frac{1}{n^2}<2-\frac{1}{n}$ $\forall\, n=2,3,\ldots$

4.2 Strong Induction

Strong Induction

To prove $P(n)$ is true $\forall\, n \in \mathbb{Z}^+$, where $P(n)$ is propositional function:

① Basis step – verify $P(1)$ is true.

② Inductive step – show $[P(1) \wedge P(2) \wedge \dots \wedge P(k)] \to P(k+1)$ true $\forall\, k \in \mathbb{Z}^+$.

Ex1 Let $P(n)$ be the statement that postage of n cents can be formed by only using 4-cent stamps and 7-cent stamps, $\forall\, n \geq 18$.

4.2 (cont)

Ex 2 Suppose a store offers gift certificates in denominations of \$25 and \$40. Determine the possible total amts you can form using these gift certificates.

4.2 (cont)

Ex 3 Find flaw w/ following "proof."

claim: For every nonnegative integer n, $5n=0$.

Pf we know $5\cdot 0=0$.

suppose $5j=0$ $\forall$ $j\in\mathbb{Z}$ w/ $0\le j\le k$.

let $k+1=i+j$ where $i,j\in\mathbb{N}$ & $i\le k, j\le k$.

By inductive hypothesis,

$5(k+1)=5(i+j)=5i+5j=0+0=0$ //

4.2 (cont)

Ex 4 Suppose $f(x)=e^x$ and $g(x)=xe^x$. Prove $g^{(n)}(x)=(x+n)e^x$.

Ex 5 Show $\sum_{j=1}^{n}\cos(jx)=\cos\left[\frac{1}{2}x(n+1)\right]\frac{\sin\left(\frac{nx}{2}\right)}{\sin(x/2)}$ $\forall\, n\in\mathbb{Z}^+$ and $\sin\left(\frac{x}{2}\right)\neq 0$.