# HW #6 Key (for graded problems)

3.4 # 5, 8, 16, 19, 24, 31 (EC)
3.5 # 3, 12, 20, 22, 28

**3.4** #5) Show if $a|b$ and $b|a$, where $a, b \in \mathbb{Z}$, then $a = b$ or $a = -b$.

<u>pf</u> $a|b \Leftrightarrow \exists\ c \in \mathbb{Z} \ni b = ca$ ①
$b|a \Leftrightarrow \exists\ d \in \mathbb{Z} \ni a = db$ ②

$\Rightarrow b = c(db)$ by substitution and since $b \neq 0$,
$\Rightarrow 1 = cd \Rightarrow c = d = 1$ or $c = d = -1$. //

#8) Prove or disprove: if $a|bc$, $a, b, c \in \mathbb{Z}^+$, then $a|b$ or $a|c$.

<u>pf</u> Not true! counter example $6 | 10(9)$ but $6 \nmid 10$ and $6 \nmid 9$.

#16) Evaluate

(a) $-17 \bmod 2 = 1$
since $-17 = -9(2) + 1$

(b) $144 \bmod 7 = 4$
since $144 = 20(7) + 4$

(c) $-101 \bmod 13 = 3$
since $-101 = 13(-8) + 3$

(d) $199 \bmod 19 = 9$
since $199 = 10(19) + 9$

#19) which of these is congruent to 5 modulo 17?

(a) $80 = 4(17) + 12 \not\equiv 5 \bmod 17$ NO
(b) $103 = 6(17) + 1 \not\equiv 5 \bmod 17$ NO
(c) $-29 = -2(17) + 5 \equiv 5 \bmod 17$ Yes
(d) $-122 = -8(17) + 14 \not\equiv 5 \bmod 17$ No

**3.4** #24) Prove if $n$ is an odd positive integer, then $n^2 \equiv 1 \pmod 8$

**Pf** $n$ is odd positive integer

$\Rightarrow \exists\, m \in \mathbb{Z}^+ \ni n = 2m+1$

$\Rightarrow n^2 = (2m+1)^2 = 4m^2 + 4m + 1$

if $m$ even, $\exists\, k \in \mathbb{Z}^+ \ni m = 2k$,

$n^2 = 4(4k^2) + 4(2k) + 1 = 16k^2 + 8k + 1 = 8(2k^2+k) + 1$

$\equiv 1 \bmod 8$

if $m$ odd, $\exists\, k \in \mathbb{Z}^+ \ni m = 2k+1$,

$n^2 = 4(4k^2+4k+1) + 4(2k+1) + 1$

$= 16k^2 + 24k + 8 + 1 = 8(2k^2+3k+1) + 1 \equiv 1 \bmod 8$ //

**3.5** #3) (a) 88 → 8, 11 $\Rightarrow 88 = 2^3 \cdot 11$

(b) 126 → 21, 6; 21 → 3, 7; 6 → 2, 3 $\quad 126 = 2 \cdot 3^2 \cdot 7$

(c) 729 → 9, 81 $\quad 729 = 3^6$

(d) 1001 → 7, 143; 143 → 11, 13 $\quad 1001 = 7 \cdot 11 \cdot 13$

(e) 1111 → 11, 101 $\quad 1111 = 11 \cdot 101$

(f) 909,090 → 9, 101010; 9 → 3, 3; 101010 → 10, 10101; 10 → 2, 5; 10101 → 7, 1443; 1443 → 3, 481; 481 → 13, 37

$909{,}090 = 2 \cdot 3^3 \cdot 5 \cdot 7 \cdot 13 \cdot 37$

3.5 #12 Are these pairwise relatively prime?

(a) 21, 34, 55
gcd (21, 34) = 1   gcd (21, 55) = 1   gcd (34, 55) = 1   yes

(b) 14, 17, 85
17(5) = 85 ⇒ No

(c) 25, 41, 49, 64
$25 = 5^2$   41 prime   ⇒ no common factors ⇒ yes
$49 = 7^2$   $64 = 2^6$

(d) 17, 18, 19, 23
17, 19 and 23 are all prime
$18 = 2 \cdot 3^2$   ⇒ no common factors ⇒ yes

#20 gcd(a, b) = ?

(a) $a = 2^2 \cdot 3^3 \cdot 5^2$   $b = 2^5 \cdot 3^3 \cdot 5^2$ ⇒ $\gcd(a,b) = 2^2 \cdot 3^3 \cdot 5^2$

(b) $a = 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13$
$b = 2^4 \cdot 3^9 \cdot 11 \cdot 17^{14}$ ⇒ $\gcd(a,b) = 2 \cdot 3 \cdot 11$

(c) $a = 17$   $b = 17^{17}$   $\gcd(a,b) = 17$

(d) $a = 2^2 \cdot 7$   $b = 5^{13} \cdot 13$ ⇒ $\gcd(a,b) = 1$

(e) $a = 0$   $b = 5$ ⇒ $\gcd(a,b) = 5$

(f) $a = 2 \cdot 3 \cdot 5 \cdot 7$
$b = 2 \cdot 3 \cdot 5 \cdot 7$ ⇒ $\gcd(a,b) = 2 \cdot 3 \cdot 5 \cdot 7$

#22 For #20, find LCM.

(a) $LCM = 2^5 \cdot 3^3 \cdot 5^2$

(b) $LCM = 2^4 \cdot 3^9 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17^{14}$

(c) $LCM = 17^{17}$

(d) $LCM = 2^2 \cdot 5^{13} \cdot 7 \cdot 13$

(e) LCM undefined

(f) $LCM = 2 \cdot 3 \cdot 5 \cdot 7$

3.5 #28) Find smallest positive integer w/ exactly $n$ different factors when $n$ is

(a) $3 = n$

factors: 1, 2, 4 $\Rightarrow$ $x = 4$

(b) $n = 4$

factors: 1, 2, 3, 6 $\Rightarrow x = 6$

(c) $n = 5$

factors: 1, 2, 4, 8, 16 $\Rightarrow x = 16$

(d) $n = 6$

$x = 2^m 3^k 5^l$ $\Rightarrow$ total # factors $= (m+1)(k+1)(l+1)$

we want $(m+1)(k+1)(l+1) = 6$

| m | k | l | x |
|---|---|---|---|
| 5 | 0 | 0 | $2^5 = 32$ |
| 2 | 1 | 0 | $2^2 \cdot 3^1 = 12$ |
| 1 | 2 | 0 | $2^1 \cdot 3^2 = 18$ |

$x = 12$

factors of 12: 1, 2, 3, 4, 6, 12

(e) $n = 10$

$x = 2^m 3^k 5^l$ total # factors $= (m+1)(k+1)(l+1)$

$(m+1)(k+1)(l+1) = 10$

| m | k | l | x |
|---|---|---|---|
| 9 | 0 | 0 | $2^9 = 512$ |
| 4 | 1 | 0 | $2^4 3 = 48$ |
| 1 | 4 | 0 | $2 \cdot 3^4 = 162$ |

$\Rightarrow x = 48$