5.1 Exponential Functions

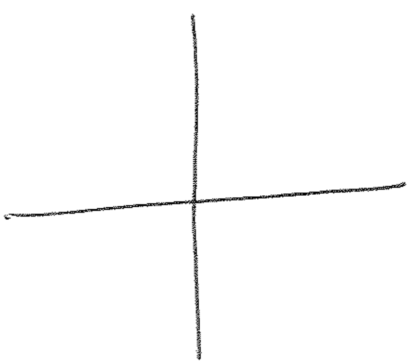
an exponential function has a variable in the exponent

Defn
If $a \in \mathbb{R}$, $a > 0$ and $a \neq 1$, then $y = f(x) = a^x$ is an exponential function with base a.

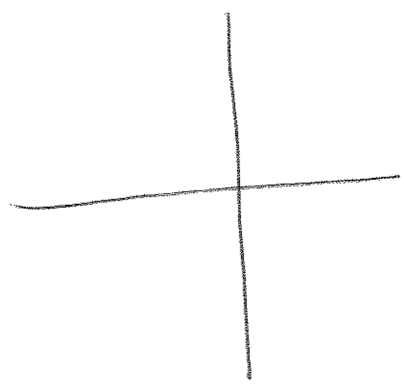
Graph of Exponential Fn

Ex1

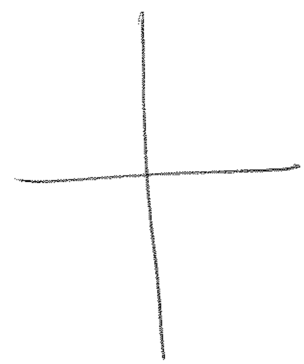
$y = 2^x$



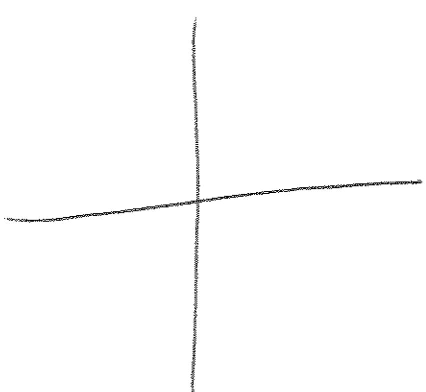
$y = 3^{-x}$



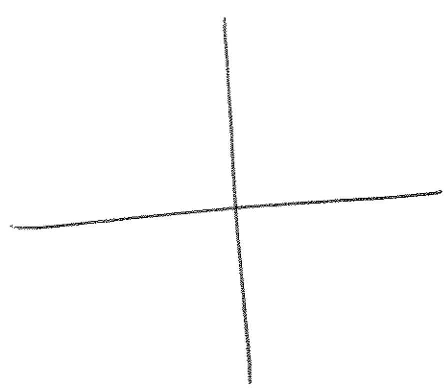
$y = \left(\frac{1}{3}\right)^x$



$y = e^{-2x}$



$y = -e^{-x}$



5.1 (cont)

Ex 2 If \$1000 is invested for x years at 10%, compounded continuously, the future value that results is $S = 1000e^{0.10x}$. What amount will result in 5 years?

Ex 3 A single bacterium splits into two bacteria every half hour, so the # of bacteria in a culture quadruples every hour. Thus the equation by which a colony of 10 bacteria multiplies in t hours is given by $y = 10(4^t)$. Graph this for $0 \le t \le 8$.

5.2 logarithmic Functions

<u>Defn</u>
For $a>0$, $a\neq 1$, the logarithmic fn $y=\log_a x$ has domain $x>0$, base a, and is defined by $a^y = x$.

i.e. $a^y = x \iff y = \log_a x$

★ the logarithm "undoes" an exponential
(like saying $a+b=c \iff c-b=a$)

<u>Ex1</u> Write $8=2^3$ in logarithmic form.

<u>Ex2</u> Rewrite $\log_3\left(\frac{1}{27}\right)=-3$ in exponential form.

5.2 (cont)

Ex 3 Evaluate

(a) $\log_5\left(\frac{1}{125}\right)$

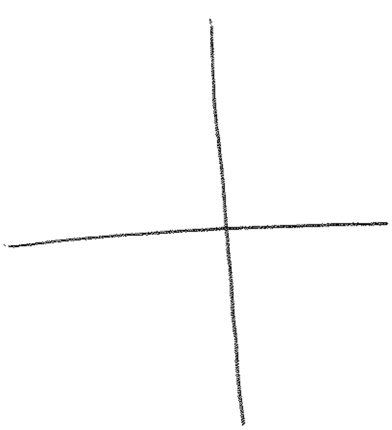
(b) $\log_7 49$

(c) $\log_2(16^{-1})$

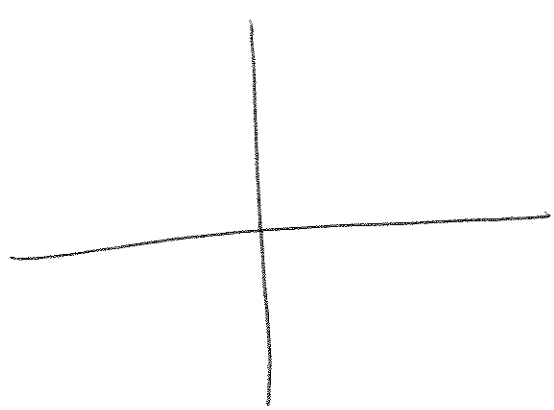
$\boxed{\text{Note: } \log \to \log_{10} \quad \text{and} \quad \ln \to \log_e}$

Graph of $y = \log_a x$

$y = \log x$



$y = -\log_3 x$



5.2 (cont)

Ex 4 Graph $y = e^x$.

log Properties ($\forall\ x \in \mathbb{R},\ a > 0,\ a \neq 1,\ m > 0,\ n > 0$)

1. $\log_a a^x = x$
2. $\log_a a = 1$
3. $\log_a 1 = 0$
4. $a^{\log_a x} = x \quad (x > 0)$
5. $\log_a (mn) = \log_a m + \log_a n$
6. $\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$
7. $\log_a (m^n) = n \log_a m$

5.2 (cont)

Proofs (5) let $u = \log_a m$ and $v = \log_a n$

$\Leftrightarrow a^u = m$ and $a^v = n$

$$\Rightarrow \log_a(mn) = \log_a(a^u a^v)$$
$$= \log_a(a^{u+v})$$
$$= u+v$$
$$= \log_a m + \log_a n \;//$$

(7) let $u = n\log_a m \Leftrightarrow \frac{u}{n} = \log_a m$

$\Leftrightarrow a^{u/n} = m$

$$\Rightarrow \log_a(m^n) = \log_a((a^{u/n})^n) = \log_a(a^u) = u$$
$$= n\log_a m \;//$$

Ex 5 Use log properties to expand these.

(a) $\ln\left(\frac{x}{x+1}\right)$

(b) $\log\left(x^2\sqrt{x-2}\right)$

5.2 (cont)

Ex 5 (cont)

(c) $\log_3 \left(\frac{y^3}{(y+4)^5} \right)$

(d) $\log_5 \left[(x+3)(x-2) \right]$

Ex 6 Write as one expression.

(a) $\log_3 8 - \frac{1}{2} \log_3 5$

(b) $2(\ln 4 - \ln 6)$

(c) $\log(2x+1) - \frac{1}{3} \log(x+1)$

5.2 (cont)

Change of Base Formula

$$\log_b x = \frac{\log_a x}{\log_a b} = \frac{\ln x}{\ln b}$$

Ex 7 Evaluate by changing the base

(a) $\log_3 12$

(b) $\log_5 17.6$

5.3 Solving logarithmic & Exponential Eqns

Exponential Eqns

Strategy:

1. isolate exponential
2. either
   (i) rewrite as log eqn (by defn)
   OR (ii) take log of both sides
3. finish getting variable by itself.

Ex 1 Solve.

(a) $4^{x+2} = 64$

(b) $2e^{x} + 3 = 13$

5.3 (cont)

Logarithmic Eqns

Ex 2 Solve.

(a) $\ln(2x-3) = \ln 11$

Strategy:

① combine log expression into single log expression (on one side of =)

② either (i) rewrite as exponential eqn (using defn)

or (ii) exponentiate both sides

③ finish solving for variable

(b) $2\log_4 x = 5$

5.3 (cont)

Ex 3 Solve.

(a) $\log_3(2x) - \log_3(x-3) = 1$

(b) $5^{x+4} - 4 = 12$

5.3 (cont)

Ex 4 Solve

(a) $2500 = 600e^{0.05x}$

(b) $3^{2x} + 3^{x} = 20$

(c) $\log(x^2) = (\log x)^2$

5.3 (cont)

Ex 5 The population of a certain city grows according to the formula $y = P_0 e^{0.03t}$. If the population was 250,000 in the year 2000, estimate the year in which the population reaches 350,000.

Ex 6 If \$1000 is invested at 10% compounded continuously, the future value S at any time t (in yrs) is given by $S = 1000e^{0.1t}$ (a) what is its worth after 1 yr? (b) How long before the investment doubles?