where x is the number of years past 1990. (*Source:* Cellular Telecommunications and Internet Association)

(a) Use a table to determine the viewing window that shows a complete graph of this model for the years from 1990 to 2010. State the window used.
(b) Graph this equation.
(c) For years beyond 2010, does this graph increase or decrease? (Look at this graph in a window that includes these years.) Explain why this means the model eventually will not accurately describe the relationship between the number of cellular subscribers and the year.

49. ***Environment*** The Millcreek watershed area was heavily strip mined for coal during the late 1960s. Because of the resulting pollution, the streams can't support fish. Suppose the cost C of obtaining stream water that contains p percent of the current pollution levels is given by

$$C = \frac{285{,}000}{p} - 2850$$

Because p is the percent of current pollution levels, $0 \le p \le 100$.

(a) Use the restriction on p and determine a range for C so that a graphing utility can be used to obtain an accurate graph. Then graph the equation.
(b) Describe what happens to the cost as p takes on values near 0.
(c) The point (1, 282,150) lies on the graph of this equation. Explain its meaning.
(d) Explain the meaning of the p-intercept.

50. ***Pollution*** Suppose the cost C of removing p percent of the particulate pollution from the exhaust gases at an industrial site is given by

$$C = \frac{8100p}{100 - p}$$

Because p is the percent particulate pollution, we know $0 \le p \le 100$.

(a) Use the restriction on p and experiment with a C-range to obtain an accurate graph of the equation with a graphing utility.
(b) Describe what happens to C as p gets close to 100.
(c) The point (98, 396,900) lies on the graph of this equation. Explain the meaning of the coordinates.
(d) Explain the meaning of the p-intercept.

51. ***Tax burden*** The dollars per capita of federal tax burden T can be described by

$$T = 3.844x^2 - 67.25x + 755.4$$

where x is the number of years past 1950. (*Source:* Internal Revenue Service, *2002 Data Book (2003)*)

(a) Graph this function for values of x that correspond to the years 1950–2010.
(b) Is the graph increasing or decreasing over these years? What does this tell us about the per capita federal tax burden?

52. ***Carbon monoxide emissions*** The number of millions of short tons of carbon monoxide emissions, y, in the United States can be described by

$$y = 0.0043x^3 - 0.24x^2 + 0.583x + 199.8$$

where x is the number of years past 1970. (*Source:* Environmental Protection Agency)

(a) Graph this function for values of x that represent the years 1970–2010.
(b) According to this function, how many short tons of emissions occurred in 2000?
(c) For years beyond 2000, does the function increase or decrease? If the model is accurate for these future years, what does this tell planners?

1.5 Solutions of Systems of Linear Equations

OBJECTIVES

- To solve systems of linear equations in two variables by graphing
- To solve systems of linear equations by substitution
- To solve systems of linear equations by elimination
- To solve systems of three linear equations in three variables

Application Preview

Suppose that a person has \$200,000 invested, part at 9% and part at 8%, and that the yearly income from the two investments is \$17,200. If x represents the amount invested at 9% and y represents the amount invested at 8%, then to find how much is invested at each rate we must find the values of x and y that satisfy both

$$x + y = 200{,}000 \quad \text{and} \quad 0.09x + 0.08y = 17{,}200$$

(See Example 3.)

Methods of solving systems of linear equations are discussed in this section.

Graphical Solution

In the previous sections, we graphed linear equations in two variables and observed that the graphs are straight lines. Each point on the graph represents an ordered pair of values (x, y) that satisfies the equation, so a point of intersection of two (or more) lines represents a solution to both (or all) the equations of those lines.

The equations are referred to as a **system of equations,** and the ordered pairs (x, y) that satisfy all the equations are the **solutions** (or **simultaneous solutions**) of the system. We can use graphing to find the solution of a system of equations.

● EXAMPLE 1 *Graphical Solution of a System*

Use graphing to find the solution of the system

$$\begin{cases} 4x + 3y = 11 \\ 2x - 5y = -1 \end{cases}$$

Solution

The graphs of the two equations intersect (meet) at the point (2, 1). (See Figure 1.33.) The solution of the system is $x = 2$, $y = 1$. Note that these values satisfy both equations.

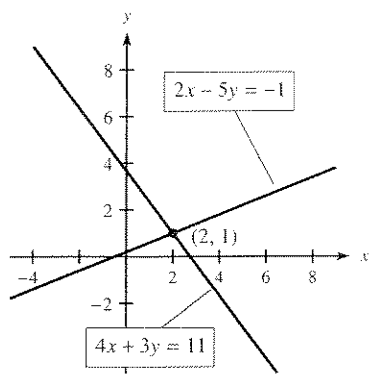


Figure 1.33

Two distinct nonparallel lines: one solution

If the graphs of two equations are parallel lines, they have no point in common and thus the system has no solution. Such a system of equations is called **inconsistent.** For example,

$$\begin{cases} 4x + 3y = 4 \\ 8x + 6y = 18 \end{cases}$$

is an **inconsistent system** (see Figure 1.34(a)).

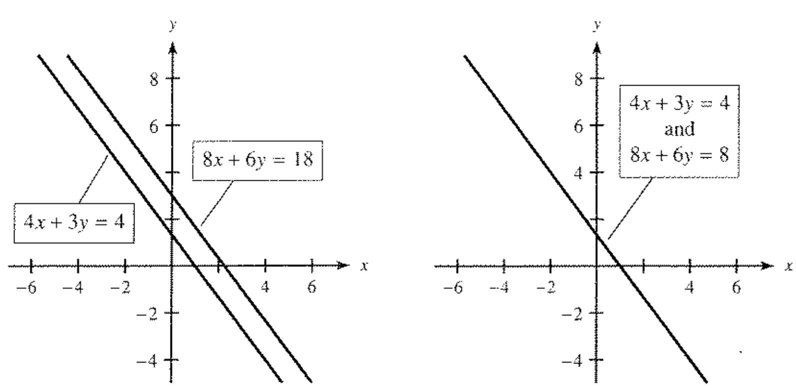


Figure 1.34

Two parallel lines: no solution

(a)

Two coincident lines: infinitely many solutions

(b)

It is also possible that two equations describe the same line. When this happens, the equations are equivalent, and values that satisfy either equation are solutions of the system. For example,

$$\begin{cases} 4x + 3y = 4 \\ 8x + 6y = 8 \end{cases}$$

is called a **dependent system** because all points that satisfy one equation also satisfy the other (see Figure 1.34(b)).

Figures 1.33, 1.34(a), and 1.34(b) represent the three possibilities that can occur when we are solving a system of two linear equations in two variables.

Solution by Substitution

Graphical solution methods may yield only approximate solutions to some systems. Exact solutions can be found using algebraic methods, which are based on the fact that equivalent systems result when any of the following operations are performed.

Equivalent Systems

Equivalent systems result when

1. One expression is replaced by an equivalent expression.
2. Two equations are interchanged.
3. A multiple of one equation is added to another equation.
4. An equation is multiplied by a nonzero constant.

The **substitution method** is based on operation (1).

Substitution Method for Solving Systems

Procedure	Example
To solve a system of two equations in two variables by substitution:	Solve the system $\begin{cases} 2x + 3y = 4 \\ x - 2y = 3 \end{cases}$
1. Solve one of the equations for one of the variables in terms of the other.	1. Solving $x - 2y = 3$ for x gives $x = 2y + 3$.
2. Substitute this expression into the other equation to give one equation in one unknown.	2. Replacing x by $2y + 3$ in $2x + 3y = 4$ gives $2(2y + 3) + 3y = 4$.
3. Solve this linear equation for the unknown.	3. $4y + 6 + 3y = 4$ $7y = -2 \Rightarrow y = -\frac{2}{7}$
4. Substitute this solution into the equation in Step 1 or into one of the original equations to solve for the other variable.	4. $x = 2\left(-\frac{2}{7}\right) + 3 \Rightarrow x = \frac{17}{7}$
5. Check the solution by substituting for x and y in both original equations.	5. $2\left(\frac{17}{7}\right) + 3\left(-\frac{2}{7}\right) = 4$ ✔ $\frac{17}{7} - 2\left(-\frac{2}{7}\right) = 3$ ✔

● **EXAMPLE 2** ***Solution by Substitution***

Solve the system

$$\begin{cases} 4x + 5y = 18 & (1) \\ 3x - 9y = -12 & (2) \end{cases}$$

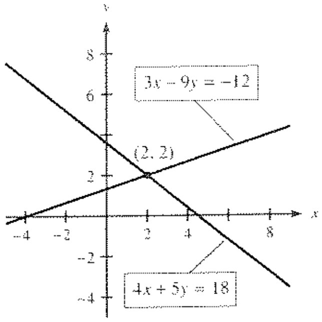


Figure 1.35

Solution

1. $x = \dfrac{9y - 12}{3} = 3y - 4$ — Solve for x in equation (2).
2. $4(3y - 4) + 5y = 18$ — Substitute for x in equation (1).
3. $12y - 16 + 5y = 18$ — Solve for y.
   $17y = 34$
   $y = 2$
4. $x = 3(2) - 4$ — Use $y = 2$ to find x.
   $x = 2$
5. $4(2) + 5(2) = 18$ and $3(2) - 9(2) = -12$ ✔ — Check.

Thus the solution is $x = 2$, $y = 2$. This means that when the two equations are graphed simultaneously, their point of intersection is (2, 2). See Figure 1.35.

Solution by Elimination

We can also eliminate one of the variables in a system by the **elimination method,** which uses addition or subtraction of equations.

Elimination Method for Solving Systems

Procedure	Example
To solve a system of two equations in two variables by the elimination method:	Solve the system $\begin{cases} 2x - 5y = 4 & (1) \\ x + 2y = 3 & (2) \end{cases}$
1. If necessary, multiply one or both equations by a nonzero number that will make the coefficients of one of the variables identical, except perhaps for signs.	1. Multiply equation (2) by −2. $2x - 5y = 4$ $-2x - 4y = -6$
2. Add or subtract the equations to eliminate one of the variables.	2. Adding gives $0x - 9y = -2$
3. Solve for the variable in the resulting equation.	3. $y = \dfrac{2}{9}$
4. Substitute the solution into one of the original equations and solve for the other variable.	4. $2x - 5\left(\dfrac{2}{9}\right) = 4$ $2x = 4 + \dfrac{10}{9} = \dfrac{36}{9} + \dfrac{10}{9}$ $2x = \dfrac{46}{9}$ so $x = \dfrac{23}{9}$
5. Check the solutions in both original equations.	5. $2\left(\dfrac{23}{9}\right) - 5\left(\dfrac{2}{9}\right) = 4$ ✔ $\dfrac{23}{9} + 2\left(\dfrac{2}{9}\right) = 3$ ✔

EXAMPLE 3 ***Investment Mix (Application Preview)***

A person has $200,000 invested, part at 9% and part at 8%. If the total yearly income from the two investments is $17,200, how much is invested at 9% and how much at 8%?

Solution

If x represents the amount invested at 9% and y represents the amount invested at 8%, then $x + y$ is the total investment,

$$x + y = 200{,}000 \qquad (1)$$

and $0.09x + 0.08y$ is the total income earned.

$$0.09x + 0.08y = 17{,}200 \qquad (2)$$

We solve these equations as follows:

$$\begin{array}{rcrll} -8x - 8y &=& -1{,}600{,}000 & (3) & \text{Multiply equation (1) by } -8. \\ 9x + 8y &=& 1{,}720{,}000 & (4) & \text{Multiply equation (2) by 100.} \\ \hline x &=& 120{,}000 & & \text{Add (3) and (4).} \end{array}$$

We find y by using $x = 120{,}000$ in equation (1).

$$120{,}000 + y = 200{,}000 \quad \text{gives} \quad y = 80{,}000$$

Thus \$120,000 is invested at 9%, and \$80,000 is invested at 8%.

As a check, we note that equation (1) is satisfied and

$$0.09(120{,}000) + 0.08(80{,}000) = 10{,}800 + 6400 = 17{,}200 \checkmark$$

● EXAMPLE 4 *Solution by Elimination*

Solve the systems:

(a) $\begin{cases} 4x + 3y = 4 \\ 8x + 6y = 18 \end{cases}$ (b) $\begin{cases} 4x + 3y = 4 \\ 8x + 6y = 8 \end{cases}$

Solution

(a) $\begin{cases} 4x + 3y = 4 \\ 8x + 6y = 18 \end{cases}$

$$\begin{array}{lrcr} \text{Multiply by } -2 \text{ to get:} & -8x - 6y &=& -8 \\ \text{Leave as is, which gives:} & 8x + 6y &=& 18 \\ \hline \text{Add the equations to get:} & 0x + 0y &=& 10 \\ & 0 &=& 10 \end{array}$$

The system is solved when $0 = 10$. This is impossible, so there are no solutions of the system. The equations are inconsistent. Their graphs are parallel lines; see Figure 1.34(a) earlier in this section.

(b) $\begin{cases} 4x + 3y = 4 \\ 8x + 6y = 8 \end{cases}$

$$\begin{array}{lrcr} \text{Multiply by } -2 \text{ to get:} & -8x - 6y &=& -8 \\ \text{Leave as is, which gives:} & 8x + 6y &=& 8 \\ \hline \text{Add the equations to get:} & 0x + 0y &=& 0 \\ & 0 &=& 0 \end{array}$$

This is an identity, so the two equations share infinitely many solutions. The equations are dependent. Their graphs coincide, and each point on this graph represents a solution of the system; see Figure 1.34(b) earlier in this section.

● EXAMPLE 5 *Medicine Concentrations*

A nurse has two solutions that contain different concentrations of a certain medication. One is a 12.5% concentration and the other is a 5% concentration. How many cubic centimeters of each should she mix to obtain 20 cubic centimeters of an 8% concentration?

Solution

Let x equal the number of cubic centimeters of the 12.5% solution, and let y equal the number of cubic centimeters of the 5% solution. The total amount of substance is

$$x + y = 20$$

and the total amount of medication is

$$0.125x + 0.05y = (0.08)(20) = 1.6$$

Solving this pair of equations simultaneously gives

$$\begin{aligned} 50x + 50y &= 1000 \\ -125x - 50y &= -1600 \\ \hline -75x \qquad &= -600 \\ x &= 8 \\ 8 + y = 20, &\text{ so } y = 12 \end{aligned}$$

Thus 8 cubic centimeters of a 12.5% concentration and 12 cubic centimeters of a 5% concentration yield 20 cubic centimeters of an 8% concentration.

Checking, we see that $8 + 12 = 20$ and

$$0.125(8) + 0.05(12) = 1 + 0.6 = 1.6 \checkmark$$

● Checkpoint

1. Solve by substitution: $\begin{cases} 3x - 4y = -24 \\ x + y = -1 \end{cases}$
2. Solve by elimination: $\begin{cases} 2x + 3y = 5 \\ 3x + 5y = -25 \end{cases}$
3. In Problems 1 and 2, the solution method is given.
   (a) In each case, explain why you think that method is appropriate.
   (b) In each case, would the other method work as well? Explain.

Graphing Utilities

We can check the solution of a system of equations in two variables or solve the system by graphing the two equations on the same screen of a graphing utility and finding a point of intersection of the two graphs. Solutions of systems of equations, if they exist, can be found by using INTERSECT. INTERSECT will give the point of intersection exactly or approximately to a large number of significant digits. ■

● EXAMPLE 6 *Solution with Technology*

Solve the system

$$\begin{cases} 3x + 2y = 12 \\ 4x - 3y = -1 \end{cases}$$

Solution

Solving both of these equations for y gives $y = 6 - \dfrac{3x}{2}$ and $y = \dfrac{1}{3} + \dfrac{4x}{3}$.

Entering the equations in a graphing utility, graphing, and using INTERSECT give the intersection of the graphs (see Figure 1.36). The point of intersection is (2, 3), so the solution of the system is $x = 2$, $y = 3$.

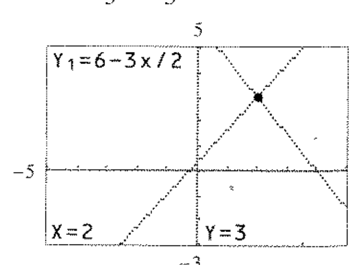


Figure 1.36

● EXAMPLE 7 *College Enrollment*

Suppose the percent of males who enrolled in college within 12 months of high school graduation is given by

$$y = -0.126x + 55.72$$

and the percent of females who enrolled in college within 12 months of high school graduation is given by

$$y = 0.73x + 39.7$$

where x is the number of years past 1960. Graphically find the year these models indicate that the percent of females will equal the percent of males.
(Source: Statistical Abstract of the United States, 2000)

Solution

Entering the two functions in a graphing utility and setting the window with x-min $= 0$, x-max $= 100$, y-min $= 0$, and y-max $= 100$ gives the graph in Figure 1.37(a). Using INTERSECT (see Figure 1.37(b)) gives a point of intersection of these two graphs at approximately $x = 18.71$, $y = 53.36$. Thus the percent of females will reach the percent of males in 1979. The models indicate that in 1979, the male percent was 53.33% and the female percent was 53.57%.

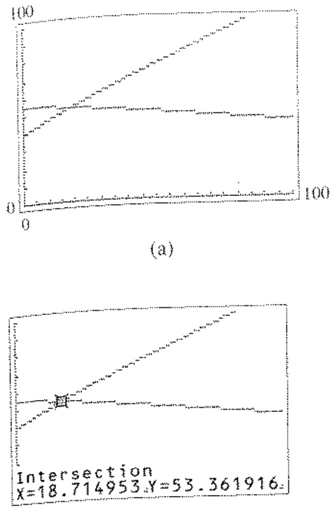
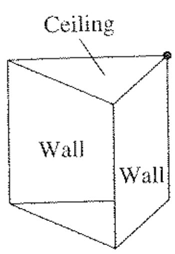


Figure 1.37

Solving a system of equations by graphing, whether by hand or with a graphing utility, is limited by two factors. (1) It may be difficult to determine a viewing window that contains the point of intersection and (2) the solution may be only approximate. With *some* systems of equations, the only practical method may be graphical approximation.

However, systems of *linear* equations can be consistently and accurately solved with algebraic methods. Computer algebra systems, some software packages (including spreadsheets), and some graphing calculators have the capability of solving systems of linear equations.



Unique solution

Figure 1.38

Three Equations in Three Variables

If a, b, c, and d represent constants, then

$$ax + by + cz = d$$

is a first-degree (linear) equation in three variables. When equations of this form are graphed in a three-dimensional coordinate system, their graphs are planes. Two different planes may intersect in a line (like two walls) or may not intersect at all (like a floor and ceiling). Three different planes may intersect in a single point (as when two walls meet the ceiling), may intersect in a line (as in a paddle wheel), or may not have a common intersection. (See Figures 1.38, 1.39, and 1.40.) Thus three linear equations in three variables may have a unique solution, infinitely many solutions, or no solution. For example, the solution of the system

$$\begin{cases} 3x + 2y + z = 6 \\ x - y - z = 0 \\ x + y - z = 4 \end{cases}$$

is $x = 1$, $y = 2$, $z = -1$, because these three values satisfy all three equations, and these are the only values that satisfy them. In this section, we will discuss only systems of three linear equations in three variables that have unique solutions. Additional systems will be discussed in Section 3.3, "Gauss-Jordan Elimination: Solving Systems of Equations."

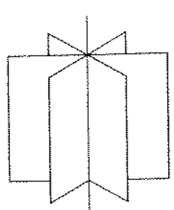

Infinitely many solutions

Figure 1.39

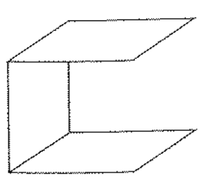

No solution

Figure 1.40

We can solve three equations in three variables using a systematic procedure called the **left-to-right elimination method.**

Left-to-Right Elimination Method

Procedure	Example
To solve a system of three equations in three variables by the left-to-right elimination method:	Solve: $\begin{cases} 2x + 4y + 5z = 4 \\ x - 2y - 3z = 5 \\ x + 3y + 4z = 1 \end{cases}$
1. If necessary, interchange two equations or use multiplication to make the coefficient of the first variable in equation (1) a factor of the other first variable coefficients.	1. Interchange the first two equations: $\begin{array}{rl} x - 2y - 3z = 5 & (1) \\ 2x + 4y + 5z = 4 & (2) \\ x + 3y + 4z = 1 & (3) \end{array}$
2. Add multiples of the first equation to each of the following equations so that the coefficients of the first variable in the second and third equations become zero.	2. Add $(-2) \times$ equation (1) to equation (2) and add $(-1) \times$ equation (1) to equation (3): $\begin{array}{rl} x - 2y - 3z = 5 & (1) \\ 0x + 8y + 11z = -6 & (2) \\ 0x + 5y + 7z = -4 & (3) \end{array}$
3. Add a multiple of the second equation to the third equation so that the coefficient of the second variable in the third equation becomes zero.	3. Add $(-\frac{5}{8}) \times$ equation (2) to equation (3): $\begin{array}{rl} x - 2y - 3z = 5 & (1) \\ 8y + 11z = -6 & (2) \\ 0y + \frac{1}{8}z = -\frac{2}{8} & (3) \end{array}$
4. Solve the third equation and *back substitute* from the bottom to find the remaining variables.	4. $z = -2$ from equation (3) $y = \frac{1}{8}(-6 - 11z) = 2$ from equation (2) $x = 5 + 2y + 3z = 3$ from equation (1) so $x = 3$, $y = 2$, $z = -2$

● EXAMPLE 8 ***Left-to-Right Elimination***

$$\text{Solve: } \begin{cases} x + 2y + 3z = 6 & (1) \\ 2x + 3y + 2z = 6 & (2) \\ -x + y + z = 4 & (3) \end{cases}$$

Solution

Using equation (1) to eliminate x from the other equations gives the equivalent system:

$$\begin{cases} x + 2y + 3z = 6 & (1) \\ -y - 4z = -6 & (2) \quad (-2) \times \text{equation (1) added to equation (2)} \\ 3y + 4z = 10 & (3) \quad \text{Equation (1) added to equation (3)} \end{cases}$$

Using equation (2) to eliminate y from equation (3) gives

$$\begin{cases} x + 2y + 3z = 6 & (1) \\ \quad -y - 4z = -6 & (2) \\ \qquad -8z = -8 & (3) \end{cases}$$ (3) × equation (2) added to equation (3)

In a system of equations such as this, the first variable appearing in each equation is called the **lead variable.** Solving for each lead variable gives

$$x = 6 - 2y - 3z$$
$$y = 6 - 4z$$
$$z = 1$$

and using **back substitution** from the bottom gives

$$z = 1$$
$$y = 6 - 4 = 2$$
$$x = 6 - 4 - 3 = -1$$

Hence the solution is $x = -1$, $y = 2$, $z = 1$.

Although other methods for solving systems of equations in three variables may be useful, the left-to-right elimination method is important because it is systematic and can easily be extended to larger systems and to systems solved with matrices. (See Section 3.3, "Gauss-Jordan Elimination: Solving Systems of Equations.")

• Checkpoint

4. Use left-to-right elimination to solve.

$$\begin{cases} x - y - z = 0 & (1) \\ \quad y - 2z = -18 & (2) \\ x + y + z = 6 & (3) \end{cases}$$

• Checkpoint Solutions

1. $x + y = -1$ means $x = -y - 1$, so $3x - 4y = -24$ becomes

$$3(-y - 1) - 4y = -24$$
$$-3y - 3 - 4y = -24$$
$$-7y = -21$$
$$y = 3$$

Hence $x = -3 - 1 = -4$, and the solution is $x = -4$, $y = 3$.

2. Multiply the first equation by (-3) and the second by (2). Add the resulting equations to eliminate the x-variable and solve for y.

$$\begin{array}{r} -6x - 9y = -15 \\ 6x + 10y = -50 \\ \hline y = -65 \end{array}$$

Hence $2x + 3(-65) = 5$ or $2x - 195 = 5$, so $2x = 200$. Thus $x = 100$ and $y = -65$.

3. (a) Substitution works well in Problem 1 because it is easy to solve for x (or for y). Substitution would not work well in Problem 2 because solving for x (or for y) would introduce fractions.

(b) Elimination would work well in Problem 1 if we multiplied the first equation by 4. As stated in (a), substitution wouldn't be as easy in Problem 2.

4. $$\begin{cases} x - y - z = 0 & (1) \\ y - 2z = -18 & (2) \\ x + y + z = 6 & (3) \end{cases}$$

Add $(-1) \times$ equation (1) to equation (3).

$$\begin{cases} x - y - z = 0 & (1) \\ y - 2z = -18 & (2) \\ 2y + 2z = 6 & (3) \end{cases}$$

Add $(-2) \times$ equation (2) to equation (3).

$$\begin{cases} x - y - z = 0 \\ y - 2z = -18 \\ 6z = 42 \end{cases}$$

Solve the equations for x, y, and z.

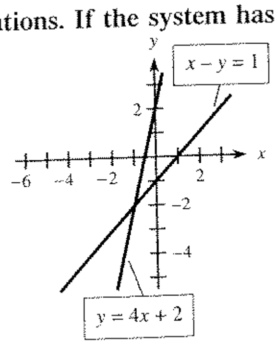
$$x = y + z$$
$$y = 2z - 18$$
$$z = 7$$

Back substitution gives $y = 14 - 18 = -4$ and $x = -4 + 7 = 3$. Thus the solution is $x = 3$, $y = -4$, $z = 7$.

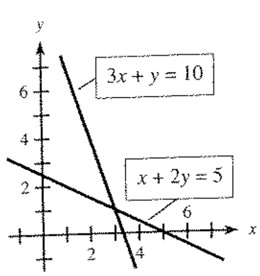
1.5 Exercises

In Problems 1–4, the graphs of two equations are shown. Decide whether the system of equations in each problem has one solution, no solution, or an infinite number of solutions. If the system has one solution, estimate it.

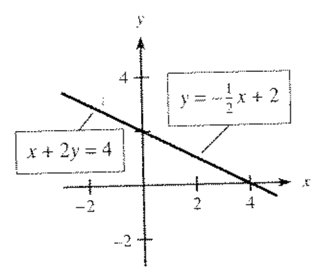
1.



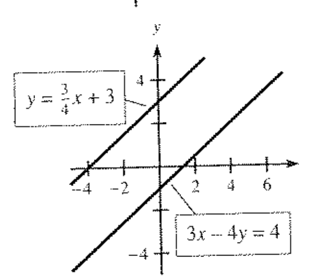
2.



3.



4.



In Problems 5–8, solve the systems of equations by using graphical methods.

5. $\begin{cases} 4x - 2y = 4 \\ x - 2y = -2 \end{cases}$

6. $\begin{cases} x - y = -2 \\ 2x + y = -1 \end{cases}$

7. $\begin{cases} 3x - y = 10 \\ 6x - 2y = 5 \end{cases}$ 8. $\begin{cases} 2x - y = 3 \\ 4x - 2y = 6 \end{cases}$

In Problems 9–12, solve the systems of equations by substitution.

9. $\begin{cases} 3x - 2y = 6 \\ 4y = 8 \end{cases}$ 10. $\begin{cases} 3x = 6 \\ 4x - 3y = 5 \end{cases}$

11. $\begin{cases} 2x - y = 2 \\ 3x + 4y = 6 \end{cases}$ 12. $\begin{cases} 4x - y = 3 \\ 2x + 3y = 19 \end{cases}$

In Problems 13–22, solve each system by elimination or by any convenient method.

13. $\begin{cases} 3x + 4y = 1 \\ 2x - 3y = 12 \end{cases}$ 14. $\begin{cases} 5x - 2y = 4 \\ 2x - 3y = 5 \end{cases}$

15. $\begin{cases} -4x + 3y = -5 \\ 3x - 2y = 4 \end{cases}$ 16. $\begin{cases} x + 2y = 3 \\ 3x + 6y = 6 \end{cases}$

17. $\begin{cases} 0.2x - 0.3y = 4 \\ 2.3x - y = 1.2 \end{cases}$ 18. $\begin{cases} 0.5x + y = 3 \\ 0.3x + 0.2y = 6 \end{cases}$

19. $\begin{cases} \frac{5}{2}x - \frac{7}{2}y = -1 \\ 8x + 3y = 11 \end{cases}$ 20. $\begin{cases} x - \frac{1}{2}y = 1 \\ \frac{2}{3}x - \frac{1}{3}y = 1 \end{cases}$

21. $\begin{cases} 4x + 6y = 4 \\ 2x + 3y = 2 \end{cases}$ 22. $\begin{cases} 6x - 4y = 16 \\ 9x - 6y = 24 \end{cases}$

Using a graphing utility or Excel to find the solution of each system of equations in Problems 23–26.

23. $\begin{cases} y = 8 - \dfrac{3x}{2} \\ y = \dfrac{3x}{4} - 1 \end{cases}$ 24. $\begin{cases} y = 9 - \dfrac{2x}{3} \\ y = 5 + \dfrac{2x}{3} \end{cases}$

25. $\begin{cases} 5x + 3y = -2 \\ 3x + 7y = 4 \end{cases}$ 26. $\begin{cases} 4x - 5y = -3 \\ 2x - 7y = -6 \end{cases}$

Using the left-to-right elimination method to solve the systems in Problems 27–32.

27. $\begin{cases} x + 2y + z = 2 \\ -y + 3z = 8 \\ 2z = 10 \end{cases}$ 28. $\begin{cases} x - 2y + 2z = -10 \\ y + 4z = -10 \\ -3z = 9 \end{cases}$

29. $\begin{cases} x - y - 8z = 0 \\ y + 4z = 8 \\ 3y + 14z = 22 \end{cases}$ 30. $\begin{cases} x + 3y - 8z = 20 \\ y - 3z = 11 \\ 2y + 7z = -4 \end{cases}$

31. $\begin{cases} x + 4y - 2z = 9 \\ x + 5y + 2z = -2 \\ x + 4y - 28z = 22 \end{cases}$ 32. $\begin{cases} x - 3y - z = 0 \\ x - 2y + z = 8 \\ 2x - 6y + z = 6 \end{cases}$

APPLICATIONS

33. *Personal expenditures* For the period 1995–2002, the total personal expenditures (in billions of dollars) in the United States for food, $f(x)$, and for housing, $h(x)$, can be described by

$$f(x) = 40.74x + 742.65 \quad \text{and} \quad h(x) = 47.93x + 725$$

where x is the number of years past 1995. (*Source:* U.S. Department of Commerce) Find the year in which these expenditures were equal and the amount spent on each.

34. *Minority children* In the United States between 1970 and 2002, the numbers of millions of black children $B(x)$ and hispanic children $H(x)$ can be described by

$$B(x) = 0.081x + 8.97 \quad \text{and} \quad H(x) = 0.282x + 3.10$$

where x is the number of years past 1970. (*Source:* U.S. Bureau of the Census)
(a) In what year were the number of children equal? How many of each were there?
(b) Do these models indicate there are more black children or hispanic children now?

35. *Pricing* A concert promoter needs to make \$42,000 from the sale of 1800 tickets. The promoter charges \$20 for some tickets and \$30 for the others.
(a) If there are x of the \$20 tickets sold and y of the \$30 tickets sold, write an equation that states that the sum of the tickets sold is 1800.
(b) How much money is received from the sale of x tickets for 20 dollars each?
(c) How much money is received from the sale of y tickets for 30 dollars each?
(d) Write an equation that states that the total amount received from the sale is 42,000 dollars.
(e) Solve the equations simultaneously to find how many tickets of each type must be sold to yield the \$42,000.

36. *Rental income* A woman has \$500,000 invested in two rental properties. One yields an annual return of 10% of her investment, and the other returns 12% per year on her investment. Her total annual return from the two investments is \$53,000. Let x represent the amount of the 10% investment and y represent the amount of the 12% investment.
(a) Write an equation that states that the sum of the investments is 500,000 dollars.
(b) What is the annual return on the 10% investment?
(c) What is the annual return on the 12% investment?
(d) Write an equation that states that the sum of the annual returns is 53,000 dollars.
(e) Solve these two equations simultaneously to find how much is invested in each property.

37. *Investment yields* One safe investment pays 10% per year, and a more risky investment pays 18% per year. A woman who has \$145,600 to invest would like to have an income of \$20,000 per year from her investments. How much should she invest at each rate?

38. *Loans* A bank lent $118,500 to a company for the development of two products. If the loan for product A was for $34,500 more than that for product B, how much was lent for each product?

39. *Rental income* A woman has $235,000 invested in two rental properties. One yields 10% on the investment, and the other yields 12%. Her total income from them is $25,500. How much is her income from each property?

40. *Loans* Mr. Jackson borrowed money from his bank and on his life insurance to start a business. His interest rate on the bank loan was 10%, and his rate on the insurance loan was 12%. If the total amount borrowed was $100,000 and his total yearly interest payment was $10,900, how much did he borrow from the bank?

41. *Nutrition* Each ounce of substance A supplies 5% of the nutrition a patient needs. Substance B supplies 12% of the required nutrition per ounce. If digestive restrictions require that the ratio of substance A to substance B be 3/5, how many ounces of each should be in the diet to provide 100% of the required nutrition?

42. *Nutrition* A glass of skim milk supplies 0.1 mg of iron and 8.5 g of protein. A quarter pound of lean red meat provides 3.4 mg of iron and 22 g of protein. If a person on a special diet is to have 7.15 mg of iron and 73.75 g of protein, how many glasses of skim milk and how many quarter-pound servings of meat would provide this?

43. *Bacterial growth* Bacteria of species A and species B are kept in a single test tube, where they are fed two nutrients. Each day the test tube is supplied with 10,600 units of the first nutrient and 19,650 units of the second nutrient. Each bacterium of species A requires 2 units of the first nutrient and 3 units of the second, and each bacterium of species B requires 1 unit of the first nutrient and 4 units of the second. What populations of each species can coexist in the test tube so that all the nutrients are consumed each day?

44. *Botany* A biologist has a 40% solution and a 10% solution of the same plant nutrient. How many cubic centimeters of each solution should be mixed to obtain 25 cc of a 28% solution?

45. *Medications* A nurse has two solutions that contain different concentrations of a certain medication. One is a 20% concentration and the other is a 5% concentration. How many cubic centimeters of each should he mix to obtain 10 cc of a 15.5% solution?

46. *Medications* Medication A is given every 4 hours and medication B is given twice each day. The total intake of the two medications is restricted to 50.6 mg per day, for a certain patient. If the ratio of the dosage of A to the dosage of B is 5 to 8, find the dosage for each administration of each medication.

47. *Pricing* A concert promoter needs to take in $380,000 on the sale of 16,000 tickets. If the promoter charges $20 for some tickets and $30 for others, how many of each type must be sold to yield the $380,000?

48. *Pricing* A nut wholesaler sells a mix of peanuts and cashews. He charges $2.80 per pound for peanuts and $5.30 per pound for cashews. If the mix is to sell for $3.30 per pound, how many pounds each of peanuts and cashews should be used to make 100 pounds of the mix?

49. *Nutrient solutions* How many cubic centimeters of a 20% solution of a nutrient must be added to 100 cc of a 2% solution of the same nutrient to make a 10% solution of the nutrient?

50. *Mixtures* How many gallons of washer fluid that is 13.5% antifreeze must a manufacturer add to 200 gallons of washer fluid that is 11% antifreeze to yield washer fluid that is 13% antifreeze?

Application Problems 51–54 require systems of equations in three variables.

51. *Nutrition* Each ounce of substance A supplies 5% of the nutrition a patient needs. Substance B supplies 15% of the required nutrition per ounce, and substance C supplies 12% of the required nutrition per ounce. If digestive restrictions require that substances A and C be given in equal amounts, and the amount of substance B be one-fifth of either of these other amounts, find the number of ounces of each substance that should be in the meal to provide 100% of the required nutrition.

52. *Dietary requirements* A glass of skim milk supplies 0.1 mg of iron, 8.5 g of protein, and 1 g of carbohydrates. A quarter pound of lean red meat provides 3.4 mg of iron, 22 g of protein, and 20 g of carbohydrates. Two slices of whole grain bread supply 2.2 mg of iron, 10 g of protein, and 12 g of carbohydrates. If a person on a special diet must have 10.5 mg of iron, 94.5 g of protein, and 61 g of carbohydrates, how many glasses of skim milk, how many quarter-pound servings of meat, and how many two-slice servings of whole grain bread will supply this?

53. *Social services* A social agency is charged with providing services to three types of clients, A, B, and C. A total of 500 clients are to be served, with $150,000 available for counseling and $100,000 available for emergency food and shelter. Type A clients require an average of $200 for counseling and $300 for emergencies, type B clients require an average of $500 for

counseling and \$200 for emergencies, and type C clients require an average of \$300 for counseling and \$100 for emergencies. How many of each type of client can be served?

54. *Social services* If funding for counseling is cut to \$135,000 and funding for emergency food and shelter is cut to \$90,000, only 450 clients can be served. How many of each type can be served in this case? (See Problem 53.)

1.6 Applications of Functions in Business and Economics

OBJECTIVES

- To formulate and evaluate total cost, total revenue, and profit functions
- To find marginal cost, revenue, and profit, given linear total cost, total revenue, and profit functions
- To write the equations of linear total cost, total revenue, and profit functions by using information given about the functions
- To find break-even points
- To evaluate and graph supply and demand functions
- To find market equilibrium

Application Preview

Suppose a firm manufactures MP3 players and sells them for \$50 each, with costs incurred in the production and sale equal to \$200,000 plus \$10 for each unit produced and sold. Forming the total cost, total revenue, and profit as functions of the quantity x that is produced and sold (see Example 1) is called the **theory of the firm.** We will also discuss **market analysis,** in which supply and demand are found as functions of price, and market equilibrium is found.

Total Cost, Total Revenue, and Profit

The **profit** a firm makes on its product is the difference between the amount it receives from sales (its revenue) and its cost. If x units are produced and sold, we can write

$$P(x) = R(x) - C(x)$$

where

$P(x)$ = profit from sale of x units

$R(x)$ = total revenue from sale of x units

$C(x)$ = total cost of production and sale of x units*

In general, **revenue** is found by using the equation

$$\text{Revenue} = (\text{price per unit})(\text{number of units})$$

The **cost** is composed of two parts, fixed costs and variable costs. **Fixed costs** *(FC)*, such as depreciation, rent, utilities, and so on, remain constant regardless of the number of units produced. **Variable costs** *(VC)* are those directly related to the number of units produced. Thus the cost is found by using the equation

$$\text{Cost} = \text{variable costs} + \text{fixed costs}$$

EXAMPLE 1 ***Cost, Revenue, and Profit (Application Preview)***

Suppose that a firm manufactures MP3 players and sells them for \$50 each. The costs incurred in the production and sale of the MP3 players are \$200,000 plus \$10 for each player produced and sold. Write the profit function for the production and sale of x players.

Solution

The total revenue for x MP3 players is $50x$, so the total revenue function is $R(x) = 50x$. The fixed costs are \$200,000, so the total cost for x players is $10x + 200{,}000$. Hence, $C(x) = 10x + 200{,}000$. The profit function is given by $P(x) = R(x) - C(x)$. Thus,

$$P(x) = 50x - (10x + 200{,}000)$$
$$P(x) = 40x - 200{,}000$$

Figure 1.41 shows the graphs of $R(x)$, $C(x)$, and $P(x)$.

*The symbols generally used in economics for total cost, total revenue, and profit are *TC*, *TR*, and π, respectively. In order to avoid confusion, especially with the use of π as a variable, we do not use these symbols.

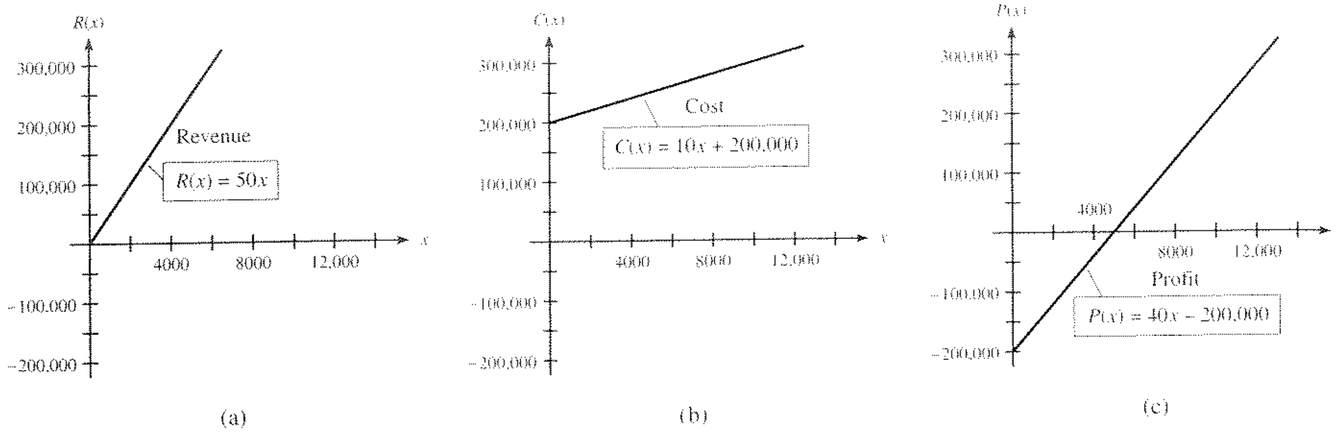


Figure 1.41

By observing the intercepts on the graphs in Figure 1.41, we note the following.

Revenue:	0 units produce 0 revenue.
Cost:	0 units' costs equal fixed costs = \$200,000.
Profit:	0 units yield a loss equal to fixed costs = \$200,000. 5000 units result in a profit of \$0 (no loss or gain).

Marginals

In Example 1, both the total revenue function and the total cost function are linear, so their difference, the profit function, is also linear. The slope of the profit function represents the rate of change in profit with respect to the number of units produced and sold. This is called the **marginal profit** ($\overline{MP}$) for the product. Thus the marginal profit for the MP3 players in Example 1 is \$40. Similarly, the **marginal cost** ($\overline{MC}$) for this product is \$10 (the slope of the cost function), and the **marginal revenue** ($\overline{MR}$) is \$50 (the slope of the revenue function).

● EXAMPLE 2 *Marginal Cost*

Suppose that the cost (in dollars) for a product is $C = 21.75x + 4890$. What is the marginal cost for this product, and what does it mean?

Solution

The equation has the form $C = mx + b$, so the slope is 21.75. Thus the marginal cost is $\overline{MC} = 21.75$ dollars per unit.

Because the marginal cost is the slope of the cost line, production of each additional unit will cost \$21.75 more, at any level of production.

Note that when total cost functions are linear, the marginal cost is the same as the variable cost. This is not the case if the functions are not linear, as we shall see later.

● Checkpoint

1. Suppose that when a company produces its product, fixed costs are \$12,500 and variable costs are \$75 per item.
   (a) Write the total cost function if x represents the number of units.
   (b) Are fixed costs equal to $C(0)$?
2. Suppose the company in Problem 1 sells its product for \$175 per item.
   (a) Write the total revenue function.
   (b) Find $R(100)$ and give its meaning.
3. (a) Give the formula for profit in terms of revenue and cost.
   (b) Find the profit function for the company in Problems 1 and 2.

● **EXAMPLE 3** ***Profit***

Suppose the profit function for a product is linear, and the marginal profit is \$5. If the profit is \$200 when 125 units are sold, write the equation of the profit function.

Solution

The marginal profit gives us the slope of the line representing the profit function. Using this slope ($m = 5$) and the point (125, 200) in the point-slope formula $P - P_1 = m(x - x_1)$ gives

$$P - 200 = 5(x - 125)$$

or

$$P = 5x - 425$$

Break-Even Analysis

We can solve the equations for total revenue and total cost simultaneously to find the point where cost and revenue are equal. This point is called the **break-even point.** On the graph of these functions, we use x to represent the quantity produced and y to represent the dollar value of revenue *and* cost. The point where the total revenue line crosses the total cost line is the break-even point.

● **EXAMPLE 4** ***Break Even***

A manufacturer sells a product for \$10 per unit. The manufacturer's fixed costs are \$1200 per month, and the variable costs are \$2.50 per unit. How many units must the manufacturer produce each month to break even?

Solution

The total revenue for x units of the product is $10x$, so the equation for total revenue is $R = 10x$. The fixed costs are \$1200, so the total cost for x units is $2.50x + 1200$. Thus the equation for total cost is $C = 2.50x + 1200$. We find the break-even point by solving the two equations simultaneously ($R = C$ at the break-even point). By substitution,

$$10x = 2.50x + 1200$$
$$7.5x = 1200$$
$$x = 160$$

Thus the manufacturer will break even if 160 units are produced per month. The manufacturer will make a profit if more than 160 units are produced. Figure 1.42 shows that for $x < 160$, $R(x) < C(x)$ (resulting in a loss) and that for $x > 160$, $R(x) > C(x)$ (resulting in a profit).

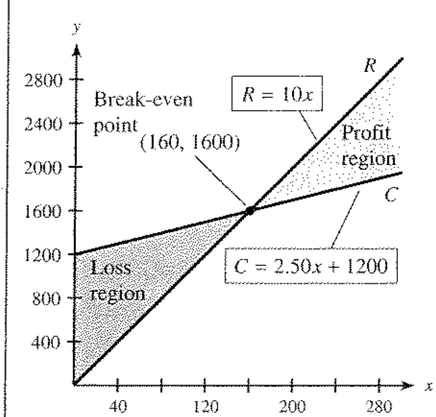


Figure 1.42

Using the fact that the profit function is found by subtracting the total cost function from the total revenue function, we can form the profit function for the previous example. The profit function is given by

$$P(x) = 10x - (2.50x + 1200) \quad \text{or} \quad P(x) = 7.50x - 1200$$

We can find the point where the profit is zero (the break-even point) by setting $P(x) = 0$ and solving for x.

$$0 = 7.50x - 1200$$
$$1200 = 7.50x$$
$$x = 160$$

Note that this is the same break-even point that we found by solving the total revenue and total cost equations simultaneously (see Figure 1.43).

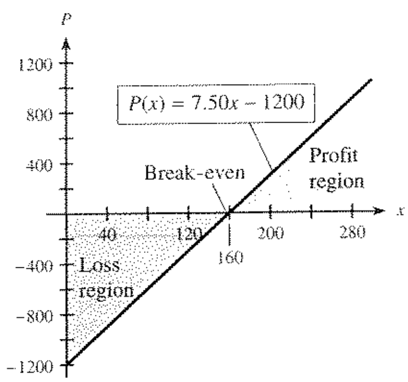


Figure 1.43

• Checkpoint

4. Identify two ways in which break-even points can be found.

Supply, Demand, and Market Equilibrium

Economists and managers also use points of intersection to determine market equilibrium. **Market equilibrium** occurs when the quantity of a commodity demanded is equal to the quantity supplied.

Demand by consumers for a commodity is related to the price of the commodity. The **law of demand** states that the quantity demanded will increase as price decreases, and that the quantity demanded will decrease as price increases. Figure 1.44 shows the graph of a typical linear demand function. Note that although quantity demanded is a function of price, economists have traditionally graphed the demand function with price on the vertical axis. Throughout this text, we will follow this tradition. Linear equations relating price p and quantity demanded q can be solved for either p or q, and we will have occasion to use the equations in both forms.

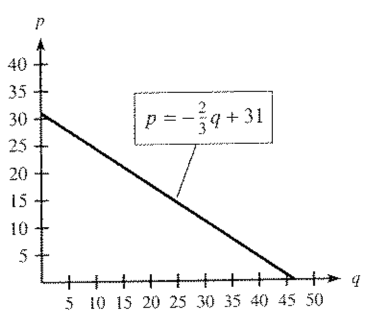


Figure 1.44

Just as a consumer's willingness to buy is related to price, a manufacturer's willingness to supply goods is also related to price. The **law of supply** states that the quantity supplied for sale will increase as the price of a product increases. Figure 1.45 shows the graph of a typical linear supply function. As with demand, price is placed on the vertical axis. Note that negative prices and quantities have no meaning, so supply and demand curves are restricted to the first quadrant.

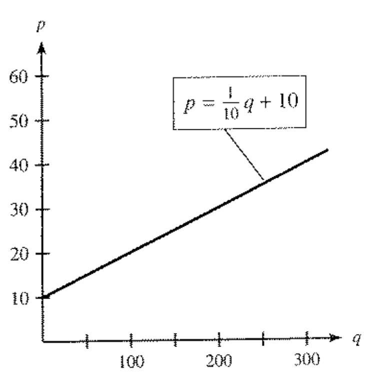


Figure 1.45

If the supply and demand curves for a commodity are graphed on the same coordinate system, with the same units, market equilibrium occurs at the point where the curves intersect. The price at that point is the **equilibrium price,** and the quantity at that point is the **equilibrium quantity.**

For the supply and demand functions shown in Figure 1.46, we see that the curves intersect at the point (30, 11). This means that when the price is \$11, consumers are willing to purchase the same number of units (30) that producers are willing to supply.

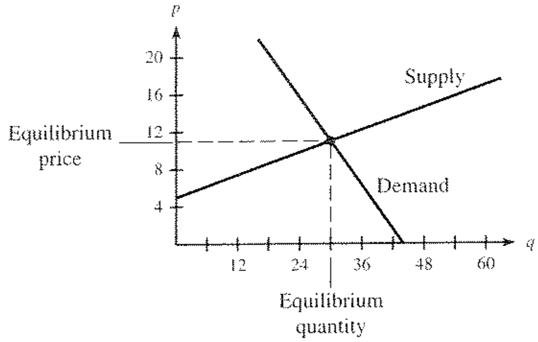


Figure 1.46

In general, the equilibrium price and the equilibrium quantity must both be positive for the market equilibrium to have meaning.

We can find the market equilibrium by graphing the supply and demand functions on the same coordinate system and observing their point of intersection. As we have seen, finding the point(s) common to the graphs of two (or more) functions is called **solving a system of equations** or **solving simultaneously.**

● EXAMPLE 5 *Market Equilibrium*

Find the market equilibrium point for the following supply and demand functions.

$$\text{Demand:}\quad p = -3q + 36$$
$$\text{Supply:}\quad p = 4q + 1$$

Solution

At market equilibrium, the demand price equals the supply price. Thus,

$$-3q + 36 = 4q + 1$$
$$35 = 7q$$
$$q = 5$$
$$p = 21$$

The equilibrium point is (5, 21).

Checking, we see that

$$21 = -3(5) + 36 ✔ \quad\text{and}\quad 21 = 4(5) + 1 ✔$$

Spreadsheet Note

We can use Goal Seek with Excel to find the market equilibrium for given supply and demand functions. To find the equilibrium quantity and price for the supply and demand functions in Example 5, we set up the table with entries for quantity, demand, and supply. We enter the functions as in the following table and add a fourth entry representing demand − supply.

	A	B	C	D
1	q	p: demand	p: supply	demand − supply
2	1	=−3*A2+36	=4*A2+1	=B2−C2

We find the equilibrium quantity and price by using Goal Seek with D2 set to 0, with changing cell A2. The resulting solution gives the equilibrium quantity as 5 and the equilibrium price as 21.

	A	B	C	D
1	q	p: demand	p: supply	demand − supply
2	5	21	21	0

● EXAMPLE 6 *Market Equilibrium*

A group of wholesalers will buy 50 dryers per month if the price is \$200 and 30 per month if the price is \$300. The manufacturer is willing to supply 20 if the price is \$210 and 30 if the price is \$230. Assuming that the resulting supply and demand functions are linear, find the equilibrium point for the market.

Solution

Representing price by p and quantity by q, we have

Demand function:

$$m = \frac{300 - 200}{30 - 50} = -5$$

$$p - 200 = -5(q - 50)$$

$$p = -5q + 450$$

Supply function:

$$m = \frac{230 - 210}{30 - 20} = 2$$

$$p - 230 = 2(q - 30)$$

$$p = 2q + 170$$

Because the prices are equal at market equilibrium, we have

$$-5q + 450 = 2q + 170$$

$$280 = 7q$$

$$q = 40$$

$$p = 250$$

The equilibrium point is (40, 250). See Figure 1.47 for the graphs of these functions.

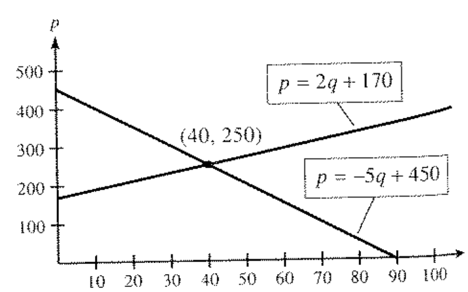


Figure 1.47

Supply and Demand with Taxation

Suppose a supplier is taxed \$$K$ per unit sold, and the tax is passed on to the consumer by adding \$$K$ to the selling price of the product. If the original supply function is $p = f(q)$, then passing the tax on gives a new supply function, $p = f(q) + K$. Because the value of the product is not changed by the tax, the demand function is unchanged. Figure 1.48 shows the effect that this has on market equilibrium.

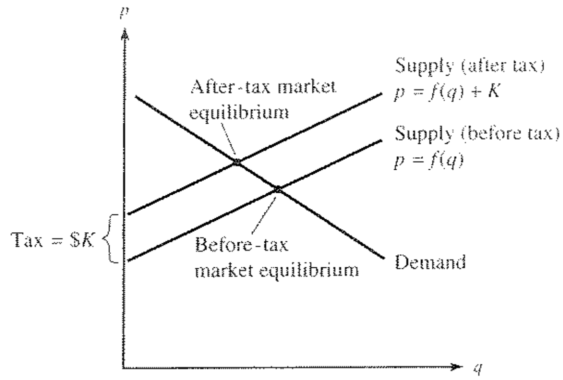


Figure 1.48

Note that the new market equilibrium point is the point of intersection of the original demand function and the new (after-tax) supply function.

● EXAMPLE 7 *Taxation*

In Example 6 the supply and demand functions for dryers were given as follows.

$$\text{Supply:} \quad p = 2q + 170$$
$$\text{Demand:} \quad p = -5q + 450$$

The equilibrium point was $q = 40$, $p = \$250$. If the wholesaler is taxed \$14 per unit sold, what is the new equilibrium point?

Solution

The \$14 tax per unit is passed on by the wholesaler, so the new supply function is

$$p = 2q + 170 + 14$$

and the demand function is unchanged. Thus we solve the system

$$\begin{cases} p = 2q + 184 \\ p = -5q + 450 \end{cases}$$
$$2q + 184 = -5q + 450$$
$$7q = 266$$
$$q = 38$$
$$p = 2(38) + 184 = 260$$

The new equilibrium point is $q = 38$, $p = \$260$.

Checking, we see that

$$260 = 2(38) + 184 \checkmark \quad \text{and} \quad 260 = -5(38) + 450 \checkmark$$

• Checkpoint

5. (a) Does a typical linear demand function have positive slope or negative slope? Why?
   (b) Does a typical linear supply function have positive slope or negative slope? Why?
6. (a) What do we call the point of intersection of a supply function and a demand function?
   (b) What algebraic technique is used to find the point named in (a)?

• Checkpoint Solutions

1. (a) $C(x) = 75x + 12{,}500$
   (b) Yes. $C(0) = 12{,}500 =$ Fixed costs. In fact, fixed costs are defined to be $C(0)$.
2. (a) $R(x) = 175x$
   (b) $R(100) = 175(100) = \$17{,}500$, which means that revenue is \$17,500 when 100 units are sold.

3. (a) Profit = Revenue − Cost or $P(x) = R(x) - C(x)$
   (b) $P(x) = 175x - (75x + 12{,}500)$
   $= 175x - 75x - 12{,}500 = 100x - 12{,}500$
4. The break-even point occurs where revenue equals cost $[R(x) = C(x)]$ or where profit is zero $[P(x) = 0]$.
5. (a) Negative slope, because demand falls as price increases.
   (b) Positive slope, because supply increases as price increases.
6. (a) Market equilibrium
   (b) Solving simultaneously

1.6 Exercises

TOTAL COST, TOTAL REVENUE, AND PROFIT

1. Suppose a calculator manufacturer has the total cost function $C(x) = 17x + 3400$ and the total revenue function $R(x) = 34x$.
   (a) What is the equation of the profit function for the calculator?
   (b) What is the profit on 300 units?
2. Suppose a stereo receiver manufacturer has the total cost function $C(x) = 105x + 1650$ and the total revenue function $R(x) = 215x$.
   (a) What is the equation of the profit function for this commodity?
   (b) What is the profit on 50 items?
3. Suppose a radio manufacturer has the total cost function $C(x) = 43x + 1850$ and the total revenue function $R(x) = 80x$.
   (a) What is the equation of the profit function for this commodity?
   (b) What is the profit on 30 units? Interpret your result.
   (c) How many radios must be sold to avoid losing money?
4. Suppose a computer manufacturer has the total cost function $C(x) = 85x + 3300$ and the total revenue function $R(x) = 385x$.
   (a) What is the equation of the profit function for this commodity?
   (b) What is the profit on 351 items?
   (c) How many items must be sold to avoid losing money?
5. A linear cost function is $C(x) = 5x + 250$.
   (a) What are the slope and the C-intercept?
   (b) What is the marginal cost, and what does it mean?
   (c) How are your answers to (a) and to (b) related?
   (d) What is the cost of producing *one more* item if 50 are currently being produced? What is it if 100 are currently being produced?
6. A linear cost function is $C(x) = 27.55x + 5180$.
   (a) What are the slope and the C-intercept?
   (b) What is the marginal cost, and what does it mean?
   (c) How are your answers to (a) and to (b) related?
   (d) What is the cost of producing *one more* item if 50 are currently being produced? What is it if 100 are currently being produced?
7. A linear revenue function is $R = 27x$.
   (a) What is the slope?
   (b) What is the marginal revenue, and what does it mean?
   (c) What is the revenue received from selling *one more* item if 50 are currently being sold? If 100 are being sold?
8. A linear revenue function is $R = 38.95x$.
   (a) What is the slope?
   (b) What is the marginal revenue, and what does it mean?
   (c) What is the revenue received from selling *one more* item if 50 are currently being sold? If 100 are being sold?
9. Let $C(x) = 5x + 250$ and $R(x) = 27x$.
   (a) Write the profit function $P(x)$.
   (b) What is the slope of the profit function?
   (c) What is the marginal profit?
   (d) Interpret the marginal profit.
10. Given $C(x) = 21.95x + 1400$ and $R(x) = 20x$, find the profit function.
    (a) What is the marginal profit, and what does it mean?
    (b) What should a firm with these cost, revenue, and profit functions do? (*Hint:* Graph the profit function and see where it goes.)
11. A company charting its profits notices that the relationship between the number of units sold, x, and the profit, P, is linear. If 200 units sold results in \$3100 profit and 250 units sold results in \$6000 profit, write the profit function for this company. Find the marginal profit.

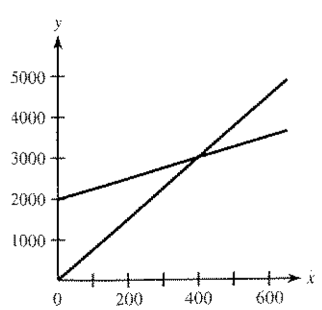
12. Suppose that the total cost function for a radio is linear, that the marginal cost is \$27, and that the total cost for 50 radios is \$4350. Write the equation of this cost function and then graph it.

13. Extreme Protection, Inc. manufactures helmets for skiing and snow boarding. The fixed costs for one model of helmet are \$6600 per month. Materials and labor for each helmet of this model are \$35, and the company sells this helmet to dealers for \$60 each.
    (a) For this helmet, write the function for monthly total costs.
    (b) Write the function for total revenue.
    (c) Write the function for profit.
    (d) Find $C(200)$, $R(200)$, and $P(200)$ and interpret each answer.
    (e) Find $C(300)$, $R(300)$, and $P(300)$ and interpret each answer.
    (f) Find the marginal profit and write a sentence that explains its meaning.

14. A manufacturer of DVD players has monthly fixed costs of \$9800 and variable costs of \$65 per unit for one particular model. The company sells this model to dealers for \$100 each.
    (a) For this model DVD player, write the function for monthly total costs.
    (b) Write the function for total revenue.
    (c) Write the function for profit.
    (d) Find $C(250)$, $R(250)$, and $P(250)$ and interpret each answer.
    (e) Find $C(400)$, $R(400)$, and $P(400)$ and interpret each answer.
    (f) Find the marginal profit and write a sentence that explains its meaning.

BREAK-EVEN ANALYSIS

15. The figure shows graphs of the total cost function and the total revenue function for a commodity.



    (a) Label each function correctly.
    (b) Determine the fixed costs.
    (c) Locate the break-even point and determine the number of units sold to break even.
    (d) Estimate the marginal cost and marginal revenue.

16. A manufacturer of shower-surrounds has a revenue function of

$$R(x) = 81.50x$$

and a cost function of

$$C(x) = 63x + 1850$$

Find the number of units that must be sold to break even.

17. A jewelry maker incurs costs for a necklace according to

$$C(x) = 35x + 1650$$

If the revenue function for the necklaces is

$$R(x) = 85x$$

how many necklaces must be sold to break even?

18. A small business recaps and sells tires. If a set of four tires has the revenue function

$$R(x) = 89x$$

and the cost function

$$C(x) = 1400 + 75x$$

find the number of sets of recaps that must be sold to break even.

19. A manufacturer sells belts for \$12 per unit. The fixed costs are \$1600 per month, and the variable costs are \$8 per unit.
    (a) Write the equations of the revenue and cost functions.
    (b) Find the break-even point.

20. A manufacturer sells watches for \$50 per unit. The fixed costs related to this product are \$10,000 per month, and the variable costs are \$30 per unit.
    (a) Write the equations of the revenue and cost functions.
    (b) How many watches must be sold to break even?

21. (a) Write the profit function for Problem 19.
    (b) Set profit equal to zero and solve for x. Compare this x-value with the break-even point from Problem 19(b).

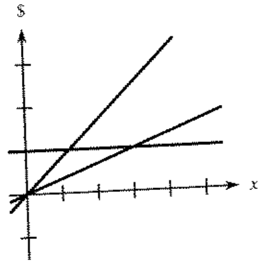
22. (a) Write the profit function for Problem 20.
    (b) Set profit equal to zero and solve for x. Compare this x-value with the break-even point from Problem 20(b).

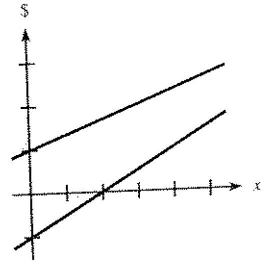
23. Electronic equipment manufacturer Dynamo Electric, Inc. makes several types of surge protectors. Their base model surge protector has monthly fixed costs of \$1045. This particular model wholesales for \$10 each and costs \$4.50 per unit to manufacture.
    (a) Write the function for Dynamo's monthly total costs.
    (b) Write the function for Dynamo's monthly total revenue.

(c) Write the function for Dynamo's monthly profit.
(d) Find the number of this type of surge protector that Dynamo must produce and sell each month to break even.

24. Financial Paper, Inc. is a printer of checks and forms for financial institutions. For individual accounts, boxes of 200 checks cost \$0.80 per box to print and package and sell for \$4.95 each. Financial Paper's monthly fixed costs for printing and packaging these checks for individuals are \$1245.
   (a) Write the function for Financial Paper's monthly total costs.
   (b) Write the function for Financial Paper's monthly total revenue.
   (c) Write the function for Financial Paper's monthly profit.
   (d) Find the number of orders for boxes of checks for individual accounts that Financial Paper must receive and fill each month to break even.
25. A company manufactures and sells bookcases. The selling price is \$54.90 per bookcase. The total cost function is linear, and costs amount to \$50,000 for 2000 bookcases and \$32,120 for 800 bookcases.
   (a) Write the equation for revenue.
   (b) Write the equation for total costs.
   (c) Find the break-even point.
26. A company distributes college logo sweatshirts and sells them for \$50 each. The total cost function is linear, and the total cost for 100 sweatshirts is \$4360, whereas the total cost for 250 sweatshirts is \$7060.
   (a) Write the equation for the revenue function.
   (b) Write the equation for the total cost function.
   (c) Find the break-even point.

In each of Problems 27 and 28, *some* of the graphs of total revenue (R), total cost (C), variable cost (VC), fixed cost (FC), and profit (P) are shown as functions of the number of units, x.
(a) Correctly label the graphs shown.
(b) Carefully sketch and label the graphs of the other functions. Explain your method.

27.


28.


SUPPLY, DEMAND, AND MARKET EQUILIBRIUM

29. As the price of a commodity increases, what happens to demand?
30. As the price of a commodity increases, what happens to supply?

Figure 1.49 is the graph of the demand function for a product, and Figure 1.50 is the graph of the supply function for the same product. Use these graphs to answer the questions in Problems 31 and 32.

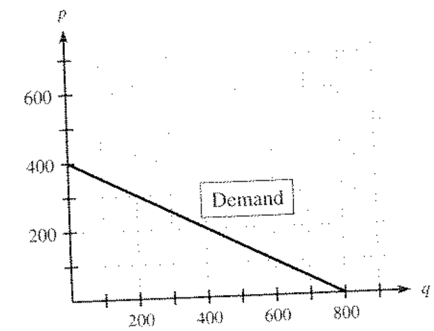


Figure 1.49

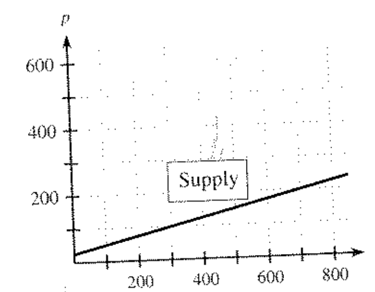
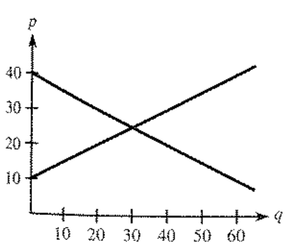


Figure 1.50

31. (a) How many units q are *demanded* when the price p is \$100?
   (b) How many units q are *supplied* when the price p is \$100?
   (c) Will there be a market surplus (more supplied) or shortage (more demanded) when $p = \$100$?
32. (a) How many units q are *demanded* when the price p is \$200?
   (b) How many units q are *supplied* when the price p is \$200?
   (c) Will there be a market surplus or shortage when the price p is \$200?

33. If the demand for a pair of shoes is given by $2p + 5q = 200$ and the supply function for it is $p - 2q = 10$, compare the quantity demanded and the quantity supplied when the price is \$60. Will there be a surplus or shortfall at this price?
34. If the demand function and supply function for Z-brand phones are $p + 2q = 100$ and $35p - 20q = 350$, respectively, compare the quantity demanded and the quantity supplied when $p = 14$. Are there surplus phones or not enough to meet demand?
35. Suppose a certain outlet chain selling appliances has found that for one brand of stereo system, the monthly demand is 240 when the price is \$900. However, when the price is \$850, the monthly demand is 315. Assuming that the demand function for this system is linear, write the equation for the demand function. Use p for price and q for quantity.
36. Suppose a certain home improvements outlet knows that the monthly demand for framing studs is 2500 when the price is \$1.00 each but that the demand is 3500 when the price is \$0.90 each. Assuming that the demand function is linear, write its equation. Use p for price and q for quantity.
37. Suppose the manufacturer of a board game will supply 10,000 games if the wholesale price is \$1.50 each but will supply only 5000 if the price is \$1.00 each. Assuming that the supply function is linear, write its equation. Use p for price and q for quantity.
38. Suppose a mining company will supply 100,000 tons of ore per month if the price is \$30 per ton but will supply only 80,000 tons per month if the price is \$25 per ton. Assuming that the supply function is linear, write its equation.

Complete Problems 39–43 by using the accompanying figure, which shows a supply function and a demand function.



39. (a) Label each function as "demand" or "supply."
    (b) Label the equilibrium point and determine the price and quantity at which market equilibrium occurs.
40. (a) If the price is \$30, what quantity is demanded?
    (b) If the price is \$30, what quantity is supplied?
    (c) Is there a surplus or shortage when the price is \$30? How many units is this surplus or shortage?
41. (a) If the price is \$20, what quantity is supplied?
    (b) If the price is \$20, what quantity is demanded?
    (c) Is there a surplus or a shortage when the price is \$20? How many units is this surplus or shortage?
42. Will a price above the equilibrium price result in a market surplus or shortage?
43. Will a price below the equilibrium price result in a market surplus or shortage?
44. Find the market equilibrium point for the following demand and supply functions.

    Demand: $p = -2q + 320$
    Supply: $p = 8q + 2$

45. Find the market equilibrium point for the following demand and supply functions.

    Demand: $2p = -q + 56$
    Supply: $3p - q = 34$

46. Find the equilibrium point for the following supply and demand functions.

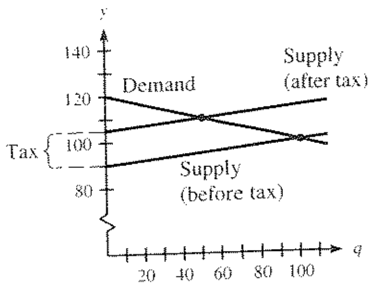
    Demand: $p = 480 - 3q$
    Supply: $p = 17q + 80$

47. Find the equilibrium point for the following supply and demand functions.

    Demand: $p = -4q + 220$
    Supply: $p = 15q + 30$

48. Retailers will buy 45 cordless phones from a wholesaler if the price is \$10 each but only 20 if the price is \$60. The wholesaler will supply 35 phones at \$30 each and 70 at \$50 each. Assuming the supply and demand functions are linear, find the market equilibrium point.
49. A group of retailers will buy 80 televisions from a wholesaler if the price is \$350 and 120 if the price is \$300. The wholesaler is willing to supply 60 if the price is \$280 and 140 if the price is \$370. Assuming the resulting supply and demand functions are linear, find the equilibrium point for the market.
50. A shoe store owner will buy 10 pairs of a certain shoe if the price is \$75 per pair and 30 pairs if the price is \$25. The supplier of the shoes is willing to provide 35 pairs if the price is \$80 per pair but only 5 pairs if the price is \$20. Assuming the supply and demand functions for the shoes are linear, find the market equilibrium point.

Problems 51–58 involve market equilibrium after taxation. Use the figure to answer Problems 51 and 52.



51. (a) What is the amount of the tax?
    (b) What is the original equilibrium price and quantity?
    (c) What is the new equilibrium price and quantity?
    (d) Does the supplier suffer from the tax even though it is passed on?
52. (a) If the tax is doubled, how many units will be sold?
    (b) Can a government lose money by increasing taxes?
53. If a \$38 tax is placed on each unit of the product of Problem 47, what are the new equilibrium price and quantity?
54. If a \$56 tax is placed on each unit of the product of Problem 46, what is the new equilibrium point?
55. Suppose that a certain product has the following demand and supply functions.

    Demand: $p = -0.05q + 65$
    Supply: $p = 0.05q + 10$

    If a \$5 tax per item is levied on the supplier and this tax is passed on to the consumer, find the market equilibrium point after the tax.
56. Suppose that a certain product has the following demand and supply functions.

    Demand: $p = -8q + 2800$
    Supply: $p = 3q + 35$

    If a \$15 tax per item is levied on the supplier, who passes it on to the consumer as a price increase, find the market equilibrium point after the tax.
57. Suppose that in a certain market the demand function for a product is given by $60p + q = 2100$ and the supply function is given by $120p - q = 540$. Then a tax of \$0.50 per item is levied on the supplier, who passes it on to the consumer as a price increase. Find the equilibrium price and quantity after the tax is levied.
58. Suppose that in a certain market the demand function for a product is given by $10p + q = 2300$ and the supply function is given by $45p - q = 360$. If the government levies a tax of \$2 per item on the supplier, who passes the tax on to the consumer as a price increase, find the equilibrium price and quantity after the tax is levied.

Key Terms and Formulas

Section	Key Terms	Formulas

1.1	Equation, members; variable; solution
	Identities; conditional equations
	Properties of equality
	Linear equation in one variable
	Fractional equation
	Linear equation in two variables
	Aligning the data
	Linear inequalities
	  Properties
	  Solutions	

1.2	Relation
	Function
	  Vertical-line test
	  Domain, range	

Section	Key Terms	Formulas
	Coordinate system	
	Ordered pair, origin, x-axis, y-axis	
	Graph	
	Function notation	
	Operations with functions	
	Composite functions	$(f \circ g)(x) = f(g(x))$
1.3	Linear function	$y = ax + b$
	Intercepts	
	x-intercept (zero of a function)	where $y = 0$
	y-intercept	where $x = 0$
	Slope of a line	$m = \dfrac{y_2 - y_1}{x_2 - x_1}$
	Rate of change	
	Parallel lines	$m_1 = m_2$
	Perpendicular lines	$m_2 = -1/m_1$
	Point-slope form	$y - y_1 = m(x - x_1)$
	Slope-intercept form	$y = mx + b$
	Vertical line	$x = a$
	Horizontal line	$y = b$
1.4	Graphing utilities	
	Standard viewing window	
	Range	
	Evaluating functions	
	x-intercept	
	Zeros of functions	
	Solutions of equations	
	x-intercept method	
1.5	Systems of linear equations	
	Solutions	
	Graphical solutions	
	Equivalent systems	
	Substitution method	
	Elimination method	
	Left-to-right elimination method	
	Lead variable	
	Back substitution	
1.6	Cost and revenue functions	$C(x)$ and $R(x)$
	Profit functions	$P(x) = R(x) - C(x)$
	Marginal profit	Slope of linear profit function
	Marginal cost	Slope of linear cost function
	Marginal revenue	Slope of linear revenue function
	Break-even point	$C(x) = R(x)$ or $P(x) = 0$
	Supply and demand functions	
	Market equilibrium	