Midterm 2 - Review Math 1320-001 (Summer 2016)

Name: ____________ Student ID Number: ____________ Score: ________

Instructions:

Total score = 100 points. There will be $5-6$ problems (but each may contain sub-problems). I suggest you take a quick glance through all the problems and do the easier ones first. Scientific calculators are allowed in the exam.

Possible topics:

1. Various convergent tests: alternating series; limit comparison; ratio; comparison tests; integral tests.

- Comparison test: If $\sum_{n=1}^{\infty} a_n$ converges, $a_n \geq b_n \geq 0$, then $\sum_{n=1}^{\infty} b_n$ converges.

- Limit comparison test: If $\lim_{n\to\infty} \frac{a_n}{b_n} = 1$, $a_n \geq 0$, $b_n \geq 0$, then $\sum_{n=1}^{\infty} a_n$ converges $\Leftrightarrow \sum_{n=1}^{\infty} b_n$ converges; $\sum_{n=1}^{\infty} a_n$ diverges $\Leftrightarrow \sum_{n=1}^{\infty} b_n$ diverges.

- Integral test: If $a_n = f(n)$, and f is a positive, decreasing, continuous function on $[1,\infty)$, then $\int_1^{\infty} f(x)\,dx < \infty \Leftrightarrow \sum_{n=1}^{\infty} a_n$ converges; $\int_1^{\infty} f(x)\,dx = \infty \Leftrightarrow \sum_{n=1}^{\infty} a_n$ diverges.

- Alternating series test: If $\sum_{n=1}^{\infty}(-1)^n b_n$ or $\sum_{n=1}^{\infty}(-1)^{n+1} b_n$ satisfies $b_n \geq b_{n+1} \geq 0$ and $\lim_{n\to\infty} b_n = 0$), then the series is convergent.

- Absolute convergence test: If $\sum_{n=1}^{\infty} |a_n|$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

- Ratio test: If $\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| > 1$, then $\sum_{n=1}^{\infty} a_n$ diverges. If $\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| < 1$, then $\sum_{n=1}^{\infty} a_n$ converges. If $\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = 1$, no conclusion is drawn.

Example. Given a series, check whether it is convergent or not, with any of these tests.

Suggestion: Review homework problems, quiz problems, and the first problem from lab 3 may also help you grasp the ideas.

2. Find power series for any given function; find the radius of convergence / interval of convergence of a power series. (Use ratio test !!!)

Example 1. Find the power series for $\sin(2x)$. (Write down the first 5 terms in the series).

Example 2. Find the radius of convergence for $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} x^n$.

3. Binomial series, geometric series.

Example 1. Find the standard power series $\sum_{n=0}^{\infty} a_n x^n$ for $f(x) = \sqrt{1+x}$.

Example 2. Find the standard power series $\sum_{n=0}^{\infty} a_n x^n$ for $f(x) = \frac{2}{1+3x}$.

4. Estimate errors of Taylor series.

Example: problem 5 from quiz 4.

5. Limit of f(x,y).

Example 1, 2, 3, 4, homework from sec 11.2, and the problem 4 from quiz 4 (solution + problem is posted).

6. Partial derivatives.

Example. Let $f(x,y) = x + y + xy$, find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, $\frac{\partial f^2}{\partial^2 x}$, $\frac{\partial f^2}{\partial x \partial y}$. Evaluate them at any given points.

7. Tangent plane.

Example. Find the tangent plane for $z = x^3 + y^2$ at point $(x, y, z) = (1, 1, 2)$.

8. Chain rule.

Example. Let $z = f(x,y) = x^2 \cdot y$, $x(t) = t^2$, $y(t) = \sin(t)$, find $\frac{dz}{dt} (= \frac{df}{dt})$.

9. Directional derivatives.

Example 1. Let $f(x,y) = x \cdot y^3$. Given unit vector $\mathbf{u} = (1, 2)$, find $D_{\mathbf{u}} f(2, 4)$.

Example 2. Let $f(x,y) = x \cdot y^3$. Which unit vector $\mathbf{u} \in \mathbb{R}^2$ will maximize $D_{\mathbf{u}} f(3, 1)$?