

Exam instructions:

Total score = 100 points. There're 7 problems. The first one is worth 16 points; for the following six problems, each problem is worth 14 points. You're allowed to use a scientific calculator during exam. I suggest you take a quick glance through all the problems and do the easier ones first.

List of formulas

*Formulas for doubling time:

- (1) new value = initial value $\times 2^{t/T_{double}}$.
- (2) Approximate doubling time: $T_{double} \approx \frac{70}{p}$, growth rate = $p\%$.
- (3) Exact doubling time: $T_{double} = \frac{\log_{10} 2}{\log_{10}(1+r)}$, growth rate = r .

*Formulas for half-life:

- (1) new value = initial value $\times (\frac{1}{2})^{t/T_{half}}$.
- (2) Approximate half-life: $T_{half} \approx \frac{70}{p}$, decay rate = $-p\%$.
- (3) Exact half-life: $T_{half} = -\frac{\log_{10} 2}{\log_{10}(1+r)}$, decay rate = r , ($r < 0$).

*General formula for a linear function:

dependent variable = initial value + (rate of change \times independent variable)

*Forms of the exponential function:

- (1) $Q = Q_0 \times (1 + r)^t$.
- (2) $Q = Q_0 \times 2^{t/T_{double}}$.
- (3) $Q = Q_0 \times (\frac{1}{2})^{t/T_{half}}$.

Topic 1, Exponential growth/decay, linear growth/decay.

A sequence is growing if the next term is greater than the previous one. Say, 1, 3, 7, 25, 99, \dots . it is decaying if the next term is less than the previous one. For example, 278, 56, 23, 5, \dots .

The sequence is growing **exponentially**, if from term to term it is multiplied by a constant which is greater than 1. Say, 1, 3, 9, 27, \dots , you can see that constant is 3. Or, $2, 2 \times (1.2), 2 \times (1.2)^2, 2 \times (1.2)^3, \dots$, the constant is 1.2. It decays **exponentially**, if from term to term it is multiplied by a constant which is less than 1. Say, $2, 1, \frac{1}{2}, \frac{1}{4}$, in this case the constant multiplied from term to term is $1/2$.

It grows **linearly**, if we're adding the same number from term to term. Say, 1, 5, 9, 13, \dots , the constant we add is 4.

To see which type the sequence is, first, look for key words. If you do not see any key words in the context, You have to judge by yourself if it is growing or decaying, and then judge it is linear,

exponential, or neither. When every time you see expressions like “grows at a rate of 5%”, it means it is an exponential sequence.

Topic 2, Half-life and doubling time.

For an exponential growth sequence, we can calculate the doubling time. For an exponential decay sequence, we may calculate the half-life. If you take a look at all the formulas we have, you can see these formulas require you to know the growth rate or decay rate.

How do you get these rates?

For example, if I say a certain group of animals has a population of 1000, and one month later it becomes 1500. If it is growing exponentially, you can see that after one month it gains 500 more animals. And these 500 animals, compared to the original 1000, is like 50%. This shows the growth rate r is 50%. Then, to use the doubling time formula, you should put $r = 50\%$ for the exact doubling time formula, and $P = 50$ for the approximate doubling time formula. Wait! In this case, $P = 50 > 15$, so the approximate doubling time does not work well.

It works similarly for calculating the half-life. But notice that the decay rate r you should put in your formula is a **negative** number.

You don't always need to calculate for half-life or doubling time with formulas. Half-life means the time it takes for a certain quantity to become half size. So if the problem is like some materials have 1000 kg, and after one year it becomes 500 kg, (assume it is decaying exponentially), what's the half-life? Well, it's just 1 year.

Topic 3, Use half-life or doubling time to predict changes.

Use the formulas 1. new value = initial value $\times 2^{t/T_{double}}$ or 2. new value = initial value $\times (\frac{1}{2})^{t/T_{half}}$.

Here t is the time difference of the initial value and the new value.

Topic 4, Functions.

There're three different ways to represent a function. You can write it in the form of a data table, in graphs, or in math formulas. You should be able to know how to convert between these expressions.

Topic 5, Linear functions.

Linear function is written in the form of $y = mx + b$. here, m means the **rate of change**, and b is the **y-intercept**, or the **initial value**.

The difficulty of linear functions is that, you do not know how to model the real life situation by a function. Usually, you have two variables, and you have to decide which one is your y (dependent variable) and what is your x (independent variable).

Once you have decided your x and y , you may then calculate your rate of change m , given by (change in y / change in x).

Once you know your rate of change m , you may plug in your data point into $y = mx + b$ to solve for b .

How do you determine which one should be your x and which one should be your y ?

Sometimes it just doesn't matter. But if the rate of change is already given to you, say something grows by \$100 dollars per minute, or 100 kg per yard, \dots , in the form of quantity A per quantity B, then A should be your y (dependent variable) and B should be your x (independent variable).

Another case is that, for example, in question 1 in quiz 6, it mentions the setup charge is \$12. that means starting at time 0, you have to pay 12 dollars. In this kind of situation, you know what's going on when one variable equals zero. This variable (time) should be your independent variable x .

To decide y , x requires some experiences. If you think one way doesn't work well, just switch and do it again.

By the way, you do not always need to use linear functions to solve real life problems. I'll show how this works in the review session.

Topic 6, Exponential functions.

Once you're familiar with doubling time and half-life, this part shouldn't be a problem for you. Identify the correct formulas to use, and in the textbook there're plots for exponential functions. So that you know how one variable changes with another variable. I suggest you read through the examples in section 9C with plots.