

Homework 3 (Markov Chains, I)  
Due Wednesday September 25, 2002

**Problem 1.** Consider the Markov chain with transition matrix

$$\mathbf{P} = \begin{vmatrix} 0.75 & 0.25 \\ 0.25 & 0.75 \end{vmatrix}.$$

Let  $\mu^n$  be the row vector  $\mu \mathbf{P}^n$ . Show by induction that

$$\mu^n = \left( \frac{1}{2} (1 + 2^{-n}), \frac{1}{2} (1 - 2^{-n}) \right).$$

What happens to  $\mu^n$  as  $n \rightarrow \infty$ . What does this say about the Markov chain?

**Problem 2.** Consider the Markov chain with transition matrix

$$\mathbf{P} = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix}.$$

and initial distribution  $\mu^0 = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ . For each  $n$ , define

$$Y_n = \begin{cases} 0 & \text{if } X_n = 1 \\ 1 & \text{otherwise.} \end{cases}$$

Show that  $(Y_0, Y_1, \dots)$  is *not* a Markov chain.

**Problem 3.** Let  $(X_0, X_1, \dots)$  be a Markov chain with transition matrix  $\mathbf{P}$ . Define  $(Y_0, \dots)$  by defining  $Y_n = X_{2n}$ . Is  $(Y_0, \dots)$  a Markov chain? If so, find its transition matrix (in terms of  $\mathbf{P}$ ).

**Problem 4.** Show that if a Markov chain is irreducible and has a state  $i$  so that  $P_{i,i} > 0$ , then the chain is also aperiodic.

Consider a chessboard with a lone white king making random moves, meaning that at each move he picks one of the possible squares to move to, uniformly at random. Is the corresponding Markov chain irreducible and/or aperiodic?

The same question, except for a bishop.

The same question, except for a knight.