

Computer Assignment 3

Math 5040 – Fall 2002

Due: Wednesday October 2 2002

Random Walk on $\mathbb{Z}/r\mathbb{Z}$

Consider the Markov chain $\{X_n\}$ on $\{0, 1, 2, \dots, r-1\}$ with

$$P_{k,j} = \begin{cases} \frac{1}{2} & \text{if } j = k \pm 1 \pmod{r} \\ 0 & \text{otherwise} \end{cases}$$

For which values of r is the chain aperiodic. Pick such a value greater than 10. Simulate the Markov chain. Does each realization of the chain converge to a value? **By simulation**, estimate the distribution of X_n for large (say larger than 10000) n . What is happening to the distribution of X_n for large values of n .

Bernoulli-Laplace Model of Diffusion

There are two urns, each always with N particles. There are a total of N white particles, and a total of N black particles. At each unit of time, a particle is chosen from each urn and interchanged.

Let X_n be the number of white particles in the first urn. Then

$$\begin{aligned} P(k, k+1) &= \left(1 - \frac{k}{N}\right)^2 \\ P(k, k-1) &= \left(\frac{k}{N}\right)^2 \\ P(k, k) &= 2\frac{k}{N} \left(1 - \frac{k}{N}\right). \end{aligned}$$

Simulate this Markov chain. For each realization of $\{X_n\}$, does it appear that X_n is converging to something? What about the distribution of X_n . Estimate via simulation the distribution of X_n for large n . What is happening for n large?