Notes on Computer Assignment 1

Math 5040 – Fall 2002

See the attached program `simexp.c` for a C program which generates 100 exponential(1) random variables and counts the fraction below 1 and above 2.

Executing this program yields the following output:

```
fraction less than 1 is 0.640000
fraction greater than 2 is 0.160000
```

Let us compare these proportions with the probabilities $\mathbb{P}\{X < 1\}$ and $\mathbb{P}\{X > 2\}$:

$$\begin{aligned}\mathbb{P}\{X < 1\} &= \int_0^1 e^{-x} \\ &= -e^{-x}\Big|_0^1 \\ &= 1 - e^{-1} \\ &= 0.632 \\ \mathbb{P}\{X > 2\} &= e^{-2} \\ &= 0.135\end{aligned}$$

Notice that we have estimated $p_1 = \mathbb{P}\{X < 1\}$ by

$$\hat{p_1} = \frac{1}{100}\sum_{i=1}^{100} \mathbf{1}\{X_i < 1\},$$

where $X_1, \ldots, X_{100}$ are i.i.d. exponential random variables.

What is the variance in our estimate?

$$\begin{aligned}\mathrm{Var}(p_1) &= \frac{1}{100^2} 100\, \mathrm{Var}(\mathbf{1}\{X_1 < 1\}) \\ &= \frac{1}{100}(0.632)(0.368) \\ &= 0.00233\end{aligned}$$

Thus the standard deviation of $\hat{p_1}$ is 0.0482. Clearly $\hat{p_1}$ has expectation equal to p_1. Thus our simulation makes sense: our estimate fell within a standard deviation of its mean.

See the attached C program `normal.c`, which generates 100 normal r.v.s by the rejection method, and counts the fraction in $(-1, 1)$.

The output of the program is

```
fraction of 100 independent normals in (-1,1) is 0.72
```

By comparison, the probability a standard normal is in $(-1, 1)$ is about 0.68.