

PROBLEM 39

We denote the cellular phones by $\{M_1, M_2, \dots, M_5\}$, the wireless phones by $\{W_1, W_2, \dots, W_5\}$ and the regular phones by $\{P_1, P_2, \dots, P_5\}$.

(a). The first ten serviced (without regard to order within the first ten) constitute a random sample without replacement of size 10 from the population of 15 phones. There are $\binom{15}{10} = 3003$ such samples. We now regard the phones as of just two types: cordless or not-cordless. The not-cordless phones are the cellular and regular phones, and there are 10 such phones. The number of ways to pick 5 cordless and 5 non-cordless is

$$(1) \quad \binom{5}{5} \binom{10}{5} = \frac{10!}{(5!)(5!)} = 252.$$

Thus the probability of getting all (five) cordless phones among the first 10 is

$$(2) \quad \frac{252}{3003} = 0.0839.$$

Here is another way of doing this: First, we figure how many orderings of the 15 phones there are with all the cordless phones among the first 10. There are $\binom{10}{5}$ possible places for the cordless phones to occupy among the first 10. [We must choose 5 out of the 10 places to put the cordless phones.] For each such selection of *locations*, there are $5!$ possible orderings of the 5 cordless phones in those locations. Once the cordless phones have been placed in the first 10 positions, there are 10 positions left over, into which must go 10 phones. There are $10!$ ways to do that. Thus, the total number of ways the cordless phones can be all placed in the first 10 positions is

$$(3) \quad \binom{10}{5} \times (5!) \times (10!).$$

There are $15!$ arrangements of the 15 phones, so the probability the cordless appear among the first 10 is

$$(4) \quad \frac{\binom{10}{5} \times (5!) \times (10!)}{15!} = 0.0839.$$

(b). Regard the last 5 phones (without regard to order within the last 5) as a sample with replacement from the 15 phones. There are $\binom{15}{5}$ such samples.

Let us count how many such samples there are containing exactly 2 out of the 3 types of phones. There are $\binom{3}{2} = 3$ ways to choose the two types. For each such choice there are $\binom{10}{5}$ possible ways to get a sample of size 5 from only these two types of phones. 2 of these samples contain all phones of the same type. Thus there

are $\binom{10}{5} - 2$ ways to pick from these two types, having both types represented. We conclude that the probability of having exactly two types in the last 5 is

$$(5) \quad \frac{3 \times [\binom{10}{5} - 2]}{\binom{15}{5}} = 0.24975.$$

(c). We want to find the probability that two phones of each type are among the first six serviced. The first six serviced can be regarded, when ignoring the order among the first 6, as an unordered sample of size 6 from 15 objects. There are $\binom{15}{6}$ such samples.

How many samples are there with two from each type. There are $\binom{5}{2}$ ways to pick two cellular phones, and $\binom{5}{2}$ ways to pick two cordless phones, and $\binom{5}{2}$ ways to pick two regular phones. Thus, there are

$$(6) \quad \binom{5}{2} \times \binom{5}{2} \times \binom{5}{2}$$

ways to pick a sample of size 6 which contains two of each type of phone. Thus the probability of selecting such a sample is

$$(7) \quad \frac{\binom{5}{2}^3}{\binom{15}{6}}.$$