TEST 1 – M3070 – Fall 2003

SHOW ALL WORK.

NAME: ________________________________

Problem 1. (10 points)

Below are the daily high temperatures, in degrees Fahrenheit, for Salt Lake City during July 2003 (31 days).

```
The decimal point is 1 digit(s) to the right of the |

   8 | 7
   9 | 12344444
   9 | 55667899
  10 | 000111222344
  10 | 55
```

Here

$$\sum_{i=1}^{31} x_i = 3038 \qquad \text{and} \qquad \sum_{i=1}^{31} x_i^2 = 298346\,.$$

(a) Find the sample mean, sample median, and sample standard deviation for this data. (5 points)

(b) Consider transforming the temperature data above into units of degrees Celsius. If F is a temperature given in units of degrees Fahrenheit, then the formula for the temperature C in units of degrees Celsius is

$$C = 0.56F - 17.78\,.$$

Find the sample mean, median, and standard deviation for the temperature expressed in units Celsius. (5 points)

Solution. (a) The mean is given by the formula

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} .$$

Here, plugging in $n = 31$ and $\sum_{i=1}^{31} x_i = 3038$ yields

$$\bar{x} = \frac{3038}{31} = 98 .$$

The sample variance is given by the short-cut formula

$$\begin{aligned} S^2 &= \frac{1}{n-1}\left[\sum_{i=1}^{n} x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2\right] \\ &= \frac{1}{30}\left(298346 - \frac{1}{31}3038^2\right) \\ &= 20.73 . \end{aligned}$$

Thus the sample standard deviation is given by

$$S = \sqrt{S^2} = 4.55 . \quad (1)$$

There are 31 observations, thus the median is given by the $32/2 = 16$th observation. This has the value 99 degrees F.

(b) If $y_i = ax_i + b$ for constants a and b, then

$$\begin{aligned} \bar{y} &= a\bar{x} + b \\ S_y &= |a| S_x \\ \tilde{y} &= a\tilde{x} + c . \end{aligned}$$

The first two we have already discussed. For the last identity, notice that the median for the $y_i's$ is still the 16th largest observation among the y_i's. But this is just the transformation $ax + b$ applied to the 16th largest observation among the x_i's.

Thus the mean, median, and standard deviation, expressed in degrees Celsius, are

$$\begin{aligned} \bar{y} &= (0.56)(98) - 17.78 = 37.1 \\ \tilde{y} &= (0.56)(99) - 17.78 = 37.66 \\ S_y &= (0.56)(4.55) = 2.548 . \end{aligned}$$

□

Problem 2. (10 points)

A sample of 26 offshore oil workers took part in a simulated escape exercise, resulting in the accompanying data on times (sec) to complete the escape:

```
The decimal point is 1 digit(s) to the right of the |

32 | 55
33 | 49
34 |
35 | 6699
36 | 34469
37 | 03345
38 | 9
39 | 2347
40 | 23
41 |
42 | 4
```

Below is some summary information:

```
 Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
325.0   359.0   369.5   370.7   391.3   424.0
```

Make a boxplot displaying this data.

Solution. In Figure 1 a boxplot of the data is drawn.

□

Problem 3. (10 points)

Suppose a "wheel-of-fortune" has 10 black numbers 1 through 10, and 10 red numbers 11 through 20. The wheel is spun twice so that each possible outcome is equally likely.

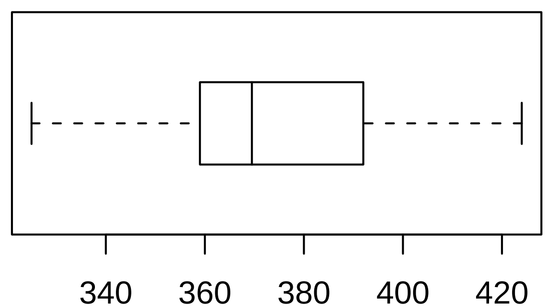

Figure 1: Boxplot of escape data

(a) Describe the sample space for this experiment.

(b) Let A be the event that a red number comes up on both spins, and let B be the event that an even number occurs on the second spin. Find $P(A), P(B)$ and $P(A \cup B)$.

Solution. (a) The outcome space $\mathcal{S}$ can be represented as

$$\mathcal{S} = \{(i, j) \ : \ 1 \leq i \leq 20, 1 \leq j \leq 20\}.$$

That is, $\mathcal{S}$ is all ordered pair whose first element represents the outcome on the first spin, and whose second element represents the outcome of the second spin. There are 20 possible outcomes for each spin, labeled 1 through 20. By the multiplication rule, there are $20 \times 20 = 400$ possible outcomes possible.

(b) Notice that $N(A)$, the number of outcomes in A, equals $10 \times 10 = 100$, since on the first spin there are 10 red numbers possible, and on the second spin there are 10 red numbers possible. Thus, $P(A) = 100/400 = 1/4$.

Likewise, $N(B) = 20 \times 10 = 200$, since any outcome is allowed on the first spin, and there are 10 even numbers on the wheel. Thus $P(B) = 200/400 = 1/2$.

Finally, $N(A \cap B)$, the number of outcomes in A *and* B, is 10×5, since there are 10 ways to get a red number on the first spin, and 5 ways to get a number which is both red and even on the second spin. Thus $P(A \cap B) = 50/400 = 1/8$.

We conclude that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{2} + \frac{1}{4} - \frac{1}{8} = \frac{5}{8}.$$

□

Problem 4. (10 points) Suppose three people each pick a number at random from 0 to 9 (without sharing any information about their choices). What is the chance that at least two of them selected the same number?

Solution. There are $10 \times 9 \times 8$ ways for 3 people to pick different numbers, and there are 10^3 ways total for 3 people to pick numbers. Thus the chance that all pick different numbers is

$$\frac{10 \times 9 \times 8}{10^3} = \frac{72}{100}.$$

Thus the chance they at least two select the same number is

$$1 - \frac{72}{100} = \frac{28}{100}.$$

□

Problem 5. (10 points)

There are 33 students registered for this class. 11 of them are women, and the remaining 22 are men.

What is the chance that if I pick 5 students at random (without replacement), that I get exactly 2 women?

Solution. There are $\binom{11}{2}$ ways to pick 2 women from 11. There are $\binom{22}{3}$ ways to pick 3 men from 22. Thus the total number of ways to pick 2 women in a sample of size 5 (implying that we also pick 3 men in the sample) is

$$\binom{11}{2}\binom{22}{3}.$$

Since there are $\binom{33}{5}$ total ways to choose 5 students from 33, the chance of getting exactly 2 women in a sample of size 5 is

$$\frac{\binom{11}{2}\binom{22}{3}}{\binom{33}{5}}.$$

□

Below is the distribution of test scores:

```
The decimal point is 1 digit(s) to the right of the |

0 | 9
1 |
1 |
2 | 3
2 | 9
3 | 23344
3 | 78889
4 | 022334
4 | 5588889
5 | 00000

 Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 9.00   35.50   42.00   40.39   48.00   50.00
```