

M3070 – FALL 2003 – Quiz 2

NAME: _____

Problem 1. There are three coins in a box. The first is a two-headed coin, the second is a fair coin (so the chance of “heads” is $1/2$), and the third is biased so that the chance of “heads” is $3/4$. When one of the coins is randomly selected and flipped, it shows heads. What is the conditional probability that it is the fair coin.

Solution. Let C_i be the event that coin i is used, where $i = 1, 2, 3$. Let H be the event that the coin which is flipped lands “heads”. Then we are given in the problem that

$$\begin{aligned}P(H | C_1) &= 1 \\P(H | C_2) &= \frac{1}{2} \\P(H | C_3) &= \frac{3}{4}.\end{aligned}$$

Since the coin is chosen at random, we have

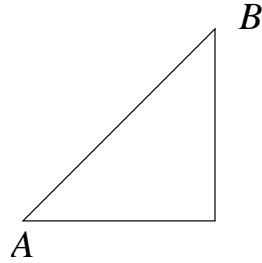
$$P(C_1) = P(C_2) = P(C_3) = \frac{1}{3}.$$

We can now calculate

$$\begin{aligned}P(C_2 | H) &= \frac{P(H \cap C_2)}{P(H)} \\&= \frac{P(H \cap C_2)}{P(H \cap C_1) + P(H \cap C_2) + P(H \cap C_3)} \\&= \frac{P(H | C_2)P(C_2)}{P(H | C_1)P(C_1) + P(H | C_2)P(C_2) + P(H | C_3)P(C_3)} \\&= \frac{(1/2)(1/3)}{(1)(1/3) + (1/2)(1/3) + (3/4)(1/3)} \\&= \frac{2}{9} \\&= 0.222.\end{aligned}$$

□

Problem 2. Suppose that I wish to travel from point A to point B in the following road system:



Each of the three road segments are either open or closed independently of each other. For each road segment, the probability that it is open is 0.8. Find the probability that I can travel from point A to point B .

Solution.

$$\begin{aligned}
 P(\text{open path}) &= 1 - P(\text{all paths closed}) \\
 &= 1 - P\left(\left\{\begin{array}{c} \text{---} \\ \text{---} \end{array} \right\} \cap \left\{\begin{array}{c} \text{---} \\ \text{---} \end{array} \right\}\right) \\
 &= 1 - P\left(\begin{array}{c} \text{---} \\ \text{---} \end{array} \text{ closed}\right) P\left(\begin{array}{c} \text{---} \\ \text{---} \end{array} \text{ closed}\right) \\
 &= 1 - \left[1 - P\left(\begin{array}{c} \text{---} \\ \text{---} \end{array} \text{ open}\right)\right] \left[1 - P\left(\begin{array}{c} \text{---} \\ \text{---} \end{array} \text{ open}\right)\right] \\
 &= 1 - \left[1 - P\left(\begin{array}{c} \text{---} \\ \text{---} \end{array} \text{ open}\right) P\left(\begin{array}{c} \text{---} \\ \text{---} \end{array} \text{ open}\right)\right] \left[1 - P\left(\begin{array}{c} \text{---} \\ \text{---} \end{array} \text{ open}\right)\right] \\
 &= 1 - [1 - (0.8)(0.8)][1 - 0.8] \\
 &= 0.928
 \end{aligned}$$

□