Math 1050-1

Spring 2004

Test 1

NAME ____

Show all your work!

Problem 1. Write the following repeated decimal as a fraction in simplest form:

$$0.24392439243924392439\ldots$$

Solution. Write

$$\begin{aligned}
x &= \frac{2439}{10000} + \frac{2439}{100000000} + \frac{2439}{1000000000000} + \cdots \\
10000x &= 2439 + \frac{2439}{10000} + \frac{2439}{100000000} + \cdots \\
10000x &= 2439 + x \\
9999x &= 2439 \\
x &= \frac{2439}{9999} \\
x &= \frac{271}{1111}
\end{aligned}$$

[8 points for 2439/9999, 10 points for 271/1111] □

Problem 2. Find all solutions to the following equation. Be sure to give an exact answer.

$$x^2 + 2x = 5\,.$$

Solution. Write the equation as

$$x^2 + 2x - 5 = 0\,, \tag{1}$$

and using the quadratic formula yields

$$\begin{aligned} x &= \frac{-2 \pm \sqrt{4 - 4(1)(-5)}}{2(1)} \\ &= \frac{-2 \pm \sqrt{24}}{2} \\ &= 1 \pm \sqrt{6} \end{aligned}$$

[Partial credit: 1 point for (1), 1 point for trying to use the quadratic formula, 2 points for correct quadratic formula] □

Problem 3. Find all x satisfying the inequality

$$x^3 - 3x^2 + 2x \geq 0\,.$$

Solution. We first find the roots of $x^3 - 3x^2 + 2x$:

$$\begin{aligned} x^3 - 3x^2 + 2x &= 0 \\ x(x^2 - 3x + 2) &= 0 \\ x(x-2)(x-1) &= 0\,. \end{aligned}$$

Thus the roots are $x = 0, 1, 2$.

Thus we have four regions to consider: $(-\infty, 0), (0, 1), (1, 2), (2, \infty)$. We will need to test the sign of the polynomial in each of these regions.

$$\begin{aligned} (-1)^3 - 3(-1)^2 + 2(-1) &= -6 \\ \left(\frac{1}{2}\right)^3 - 3\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right) &= \frac{3}{8} \\ \left(\frac{3}{2}\right)^3 - 3\left(\frac{3}{2}\right)^2 + 2\left(\frac{3}{2}\right) &= -\frac{3}{8} \\ (3)^3 - 3(3)^2 + 2(3) &= 6\,. \end{aligned}$$

Thus, the polynomial is non-negative in the intervals $[0, 1]$ and $[2, \infty)$.

[Partial credit: 2 points for trying to find roots, 2 points for testing in intervals between roots.]

□

Problem 4. Write

$$\frac{1}{x} - \frac{1}{x+1}$$

as a rational expression (the quotient of two polynomials). What is the *domain* of this expression?

Solution.

$$\begin{aligned}\frac{1}{x} - \frac{1}{x+1} &= \frac{x+1}{x(x+1)} - \frac{x}{x(x+1)} \\ &= \frac{x+1-x}{x(x+1)} \\ &= \frac{1}{x(x+1)}\,.\end{aligned}$$

The domain is all x *except* $x = 0, -1$.

[partial credit: 7 points for writing as rational expression, 3 points for finding domain.] □

Problem 5. Write the following polynomial in standard form, and find the degree and constant term:

$$(x^2 - 2)(-2x + x^3 + 1)$$

Solution. We have

$$\begin{aligned}(x^2 - 2)(-2x + x^3 + 1) &= -2x^3 + x^5 + x^2 + 4x - 2x^3 - 2 \\ &= x^5 - 4x^3 + x^2 - 2\,.\end{aligned}$$

Thus the degree is 5 and the constant term is -2.

[partial credit: 6 points for standard form, 2 points for the degree, and 2 points for the constant term.] □

Problem 6. Solve the following equation:

$$\frac{x-2}{x+1} = \frac{x-5}{x+2}$$

Solution.

$$\begin{aligned}
\frac{x-2}{x+1} &= \frac{x-5}{x+2} \\
x-2 &= \frac{x-5}{x+2}(x+1) \\
(x+2)(x-2) &= (x-5)(x+1) \qquad (2) \\
x^2-4 &= x^2-4x-5 \\
-4 &= -4x-5 \\
x &= -\frac{1}{4}
\end{aligned}$$

□

[partial credit: 3 points for (2).]

Problem 7. You invest money at an (unusually good) annual interest rate that will triple your investment every 2 years. What is the interest rate?

Solution. We use the formula that the amount of money after t years with an initial investment of P is $A = P(1+r)^t$.

We solve for r:

$$\begin{aligned}
3P &= P(1+r)^2 \qquad (3) \\
3 &= (1+r)^2 \\
\sqrt{3} &= 1+r \\
r &= \sqrt{3}-1 \\
r &\approx 0.732
\end{aligned}$$

Thus $r = 73.2\%$. □

[partial credit: 4 points for setting up the equation (3).]

Problem 8. If

$$\frac{x^a}{y^b} = \left(\frac{x^{-1/3}y^2}{yx^{2/3}}\right)^{1/2},$$

then what are a and b?

Solution.

$$\begin{aligned}\left(\frac{x^{-1/3}y^2}{yx^{2/3}}\right)^{1/2} &= \left(x^{-1/3-2/3}y^{2-1}\right)^{1/2} \\ &= \left(x^{-1}y\right)^{1/2} \\ &= x^{-1/2}y^{1/2} \\ &= \frac{x^{-1/2}}{y^{-1/2}}.\end{aligned}$$

Thus $a = b = -1/2$.

[partial credit: 5 points for either a or b.] □

Problem 9. Solve the following equation:

$$|2x - 1| = x + 1.$$

Solution. We have two equations. The first is:

$$\begin{aligned}2x - 1 &= x + 1 \\ x &= 2.\end{aligned}$$

The second is

$$\begin{aligned}2x - 1 &= -(x + 1) \\ 3x &= 0 \\ x &= 0.\end{aligned}$$

Thus the solutions are $x = 0$ and $x = 2$.

[partial credit: 4 points for only one correct solution.] □

Distribution of Test Scores

```
The decimal point is 1 digit(s) to the right of the |

0 | 777
1 |
1 | 778
2 | 122
2 | 556667799
3 | 11133444
3 | 55777999
4 | 0000011113333334444
4 | 5556666888999
5 | 0000011234444
5 | 5666677777889999
6 | 00011111233334
6 | 5555555666666677777888888899 9
7 | 00111334444
7 | 555666689
8 | 001123
8 | 57


 Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 7.00   41.00   56.00   53.45   67.00   87.00
```