

Math 1050-1
Spring 2004
Test 2

NAME _____

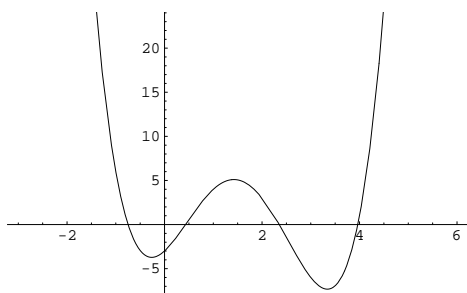
Show all your work!

Problem 1. In each of the following graphs, the horizontal axis is the x -axis, and the vertical axis is the y -axis. Say for each of the following graphs

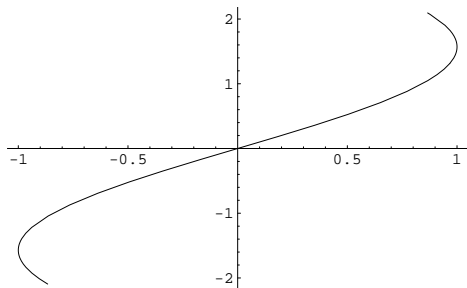
(i) If it is the graph of a function of the variable x .

(ii) If it is the graph of a function f of the variable x , does the inverse function f^{-1} exist?

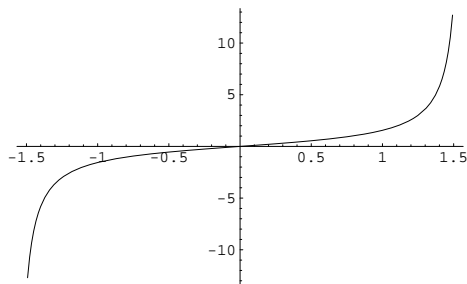
(a)



(b)



(c)



Solution. [10 points total]

- (i) This graph passes the vertical line test, so it is the graph of a function of x . [2 points]
It fails the horizontal line test, so it is not a one-to-one function, and hence the inverse does not exist. [2 points]
- (ii) This graph fails the vertical line test, so it is not the graph of a function of x . [2 points]
- (iii) This graph passes the vertical line test, so it is a function of x . [2 points]. It passes the horizontal line test, so it is one-to-one, and hence the inverse exists. [2 points].

[5 points for getting the axis mixed up, and correct answers for reversed axis, provided there is some indication of an explanation.] □

Problem 2. Let $f(x) = \frac{1}{x-5}$ and $g(r) = \sqrt{r-3}$. Give a formula, in as simple form as possible, for each of the following functions, and find the domain for each:

(i) $(f \circ g)(x)$.

(ii) $(g \circ f)(x)$.

(iii) $(gf)(x)$.

Solution. [Grading: 2 points for each correct function and domain, so 12 points total.]

(i)

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(\sqrt{x-3}) \\ &= \frac{1}{\sqrt{x-3}-5}.\end{aligned}$$

The domain is all x so that

(a) $x - 3 \geq 0$,

(b) $\sqrt{x-3} - 5 \neq 0$.

Thus we need all x so that $x \geq 3$, excluding x so that

$$\begin{aligned}\sqrt{x-3} - 5 &= 0 \\ \sqrt{x-3} &= 5 \\ x - 3 &= 25 \\ x &= 28.\end{aligned}$$

Thus the domain is all x so that $x \geq 3$, excluding $x = 28$.

(ii)

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g\left(\frac{1}{x-5}\right) \\ &= \sqrt{\frac{1}{x-5} - 3}.\end{aligned}$$

x is in the domain if

$$\begin{aligned}\frac{1}{x-5} - 3 &\geq 0 \\ \frac{1}{x-5} &\geq 3.\end{aligned}\tag{1}$$

Suppose that $x < 5$. Then $\frac{1}{x-5} < 0$, so x cannot satisfy the inequality (1). If $x > 5$, then $x - 5 > 0$, and we can multiply both sides of the inequality (1) by $x - 5$ to get

$$\begin{aligned}1 &\geq 3(x-5) \\ 1 &\geq 3x - 15 \\ 16 &\geq 3x \\ \frac{16}{3} &\geq x.\end{aligned}$$

Thus the only x satisfying (1) are $5 < x \leq \frac{16}{3}$. Thus, the domain of the function is the interval $(5, \frac{16}{3}]$.

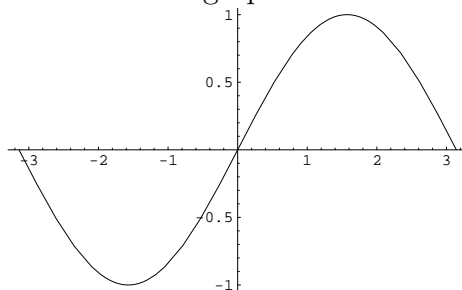
(iii) We have

$$\begin{aligned}(fg)(x) &= \frac{1}{x-5}\sqrt{x-3} \\ &= \frac{\sqrt{x-3}}{x-5}.\end{aligned}$$

The domain here is all x so that $x \geq 3$, except $x = 5$.

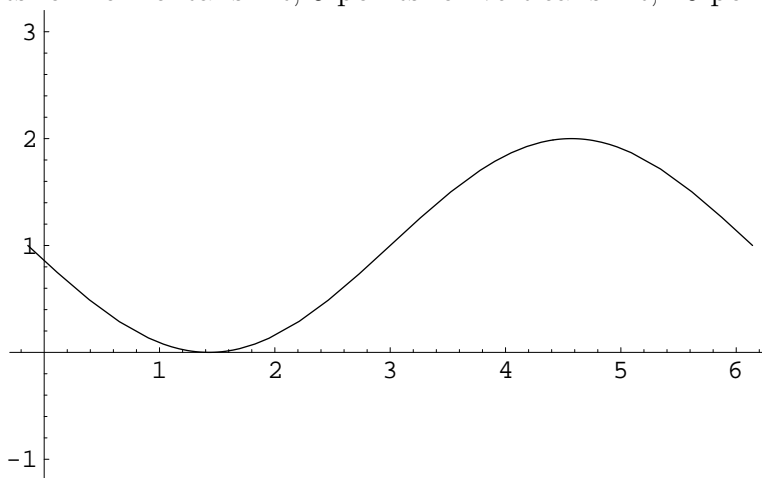
□

Problem 3. Let $f(x)$ be the function with graph below:



Graph the function $h(x) = f(x - 3) + 1$.

Solution. [5 points for horizontal shift, 5 points for vertical shift, 10 points total.]



□

Problem 4. Consider the equation

$$x^{10} - y^{21} + \sqrt{2} - 5 = 0.$$

(i) Is x a function of y ?

(ii) Is y a function of x ?

Solution. [2 point each for correct answer without explanation. 5 point each for correct with explanation.]

(i) Isolating x gives

$$x^{10} = y^{21} + \sqrt{2} - 5.$$

This has two solutions:

$$x = \left(y^{21} + \sqrt{2} - 5\right)^{1/10}$$
$$x = -\left(y^{21} + \sqrt{2} - 5\right)^{1/10}.$$

Thus for a given value of y , there is not a unique value of x so that (x, y) is a solution. That is, x is not a function of y .

(ii) Isolating y on one side of the equation gives

$$y^{21} = x^{10} - \sqrt{2} + 5.$$

When n is odd, $(a^n)^{1/n} = a$, so we can take the 21st root on both sides above to get

$$y = \left(x^{10} - \sqrt{2} + 5\right)^{1/21}.$$

Thus y is uniquely determined by x , so y is a function of x . □

Problem 5. Consider the function

$$f(x) = \frac{10x - \sqrt{7}}{3x + \sqrt{2}}.$$

- (i) What is the domain of f ?
- (ii) What is the range of f ?
- (iii) If the inverse function f^{-1} exists, find it.

Solution. [10 points]

- (i) The domain of f is all x except where

$$\begin{aligned} 3x + \sqrt{2} &= 0 \\ x &= \frac{-\sqrt{2}}{3}. \end{aligned}$$

- (ii) To find the range, we check for which y we can solve the equation $y = f(x)$ for (at least one) x .

$$\begin{aligned} \frac{10x - \sqrt{7}}{3x + \sqrt{2}} &= y \\ 10x - \sqrt{7} &= y(3x + \sqrt{2}) \\ 10x - \sqrt{7} &= 3xy + \sqrt{2}y \\ 10x - 3xy &= \sqrt{2}y + \sqrt{7} \\ x(10 - 3y) &= \sqrt{2}y + \sqrt{7} \\ x &= \frac{\sqrt{2}y + \sqrt{7}}{10 - 3y}. \end{aligned}$$

This solution makes sense provided $10 - 3y \neq 0$, that is, for $y \neq \frac{10}{3}$. Thus the range is all y excluding $y = \frac{10}{3}$.

- (iii) The formula for x above in terms of y gives the inverse function:

$$f^{-1}(y) = \frac{\sqrt{2}y + \sqrt{7}}{10 - 3y}.$$

□

Problem 6. Divide the polynomial $3x^4 - 2x^2 + 5x - 1$ by the polynomial $x^2 + 3$.

Solution. [10 points]

		$3x^2$	$+0x$	-11
$x^2 + 0x + 3$	$3x^4$	$+0x^3$	$-2x^2$	$+5x$
	$3x^4$	$+0x^3$	$+9x^2$	-1
		$+0x^3$	$-11x^2$	$+5x$
		$+0x^3$	$+0x^2$	$+0x$
			$-11x^2$	$+5x$
			$-11x^2$	$+0x$
				-33
				$5x$
				$+32$

Thus

$$\frac{3x^4 - 2x^2 + 5x - 1}{x^2 + 3} = 3x^2 - 11 + \frac{5x + 32}{x^2 + 3}.$$

□

Problem 7. Consider the line ℓ given by the equation $y = -3x + 10$. Find the equation for the line perpendicular to ℓ going through the point $(1, 2)$.

Solution. [10 points]

The perpendicular line has slope $-1/(-3) = 1/3$. In point-slope form, we have

$$(y - 2) = \frac{1}{3}(x - 1).$$

We can rearrange to get in slope-intercept form:

$$y = 2 + \frac{1}{3}x - \frac{1}{3}$$
$$y = \frac{1}{3}x + \frac{5}{3}.$$

□

Problem 8. Suppose the height of a rocket is described by the equation

$$h(t) = 10 + 1000t - 16t^2,$$

where $h(t)$ is in the units of feet, and t is in units of seconds.

- (i) How long is the rocket in the air?
- (ii) What is its maximum height?

Solution. [10 points]

- (i) We need to solve $h(t) = 0$. We use the quadratic formula:

$$\begin{aligned} t &= \frac{-1000 \pm \sqrt{1000^2 - 4(-16)(10)}}{-32} \\ &= \frac{-1000 \pm 8\sqrt{15635}}{-32} \\ &= \frac{125}{4} \pm \frac{\sqrt{15635}}{4}. \end{aligned}$$

We want the larger root, so we have

$$t = \frac{125 + \sqrt{15635}}{4}.$$

- (ii) We can complete the square:

$$\begin{aligned} h(t) &= -16t^2 + 1000t + 10 \\ &= -16 \left(t^2 - \frac{1000}{16}t \right) + 10 \\ &= -16 \left[t^2 - 2\frac{1000}{32}t + \left(\frac{1000}{32} \right)^2 - \left(\frac{1000}{32} \right)^2 \right] + 10 \\ &= -16 \left(t - \frac{1000}{32} \right)^2 + 16 \left(\frac{1000}{32} \right)^2 + 10. \end{aligned}$$

Thus the graph of h is a downward facing parabola with vertex at $t = \frac{1000}{32}$ and of height

$$16 \left(\frac{1000}{32} \right)^2 + 10 = 15635.$$

□