Students' ownership of their learning and their mathematics

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University of Utah, Teaching seminar
February 14th, 2020

Starting observation: some context…

- Math 2200, Discrete Mathematics
- Class of less than 20 students
- Group work on challenging problems

This means:

- First proof-based class
- Students
  - knew each other
  - used to group work
  - used to the worksheets being challenging

I observed that the students:

- modified the problems from the worksheets
- explored further
- created their own problems

Without any instruction to do so!

How many 4-permutations of the positive integers not exceeding 100 contain three consecutive integers $k, k+1, k+2$ in the correct order

(a) where they are in consecutive positions in the permutation?

(b) where these consecutive integers can perhaps be separated by other integers in the permutation?

Students' versions:

- What if these 3 integers don't need to be in order?
- What about 5-permutations with the same condition?
- What if we only consider integers smaller than 50?

My questions after this observation:

1. What consequences would this process have on the students' learning, if any?

2. What was it, from the material or course structure, that inspired the students to be in control of their mathematics?

Students were taking ownership of their learning and mathematics!

But what does this mean?

What does students' ownership of learning mean?

Consists of five components (following Conley & French, 2014):

1. Motivation and Engagement
2. Goal orientation and Self-direction
3. Self-efficacy and Self-confidence
4. **Metacognition and Self-monitoring**
5. Persistence

This can be explicitly **taught** to all students

Conley & French, 2014

Motivation:

- Extrinsic
- **Intrinsic**

Engagement:

- Behavioral (norms, expectations)
- Emotional (enjoyment, interest)
- **Cognitive (investment in learning)**

Goal orientation and self-direction.

- Setting goals
- Identifying resources
- Identifying steps

Goal needs to be achievable.

High goal-orientation leads to growth mindset.

Self-efficacy is defined as

"students' confidence in their ability to complete increasingly challenging and complex academic and career tasks and be able to build on past experiences and success to maximise future successes"

(Conley, 2014)

Self-efficacy is

- Specific to a context
- Specific to a task

(different from self-confidence)

Response to academic obstacles: new strategies and more effort. (Dweck et al., 2011)

Metacognition and self-monitoring

- Awareness of learning
- Actively engaged in learning
- Reflection on the learning process
- Development of new strategies

The key component of ownership of learning!

Persistence

- Sustained hard work toward a goal
- Awareness of obstacles
- Seek help as needed
- Value hard work but also “smart” work (Dweck et al., 2011)

What does students' ownership of mathematics mean?

Doing mathematics instead of **applying** methods.

Let students

- (Re)-discover formulas/theorems/results
- Use their own words
- Get to a conclusion through various paths

Closer to what research mathematicians do!

“Someone else may have done it already in a book, but I just don’t understand it unless I try it myself and put it in my own terms”

(student quote in Maher, 2005)

When students become owners of their learning and their mathematics, what are the (positive) consequences?

Consider the consequences on

- Motivation
- Achievement
- Diversity

Consequences on motivation

Ownership of **mathematics** increases students' intrinsic motivation!

- By working on their own problems
  - More natural questions
- By getting a stronger sense of achievement
- By becoming driven by a desire to make sense (Maher, 2005)

It gives students responsibilities

- They might choose challenges/difficult problems on their own
- Successes are fully their own

Different approaches:

- Students want to convince their peers

It opens the door to (Maher, 2005):

- Extensions of original problem
- New investigations

It motivates the students to go further.

Consequences on achievement

Ownership of learning model predicts student achievement such as (Richardson et al., 2012, Dweck et al., 2011, among others):

- standardized test scores
- high school grades
- college and graduate school GPA
- college retention

At all levels!

- Indicator of college readiness (Conley & French, 2014)
- Helps students succeed in more diverse environments
  - Including non-academic setting
- Leads to a more conceptual understanding of mathematics (Mueller et al., 2011)
  - Impacts grades and skills

Consequences on diversity

- More students feel like they belong
  - By having space for different ways of thinking / other approaches
  - By bringing a creative aspect to mathematics
- Increases student empowerment (Gay, 2000)
- Helps students develop their own mathematical identity (Gay, 2000)

Some of my (naive) hypotheses:

- If students **do** mathematics instead of **copying** someone else's mathematics, are role models becoming less essential?
- **Who** discovered which result becomes less important.
    - More students might see themselves as mathematicians.

Example used in my linear algebra class

(thank you Kelly for suggesting this!)

How do research mathematicians work?

- Collaborative work
- Decide if a result is valid using
  - Standards within the mathematics community
  - Peer review

My goal: give my students a similar experience with mathematics (proof-writing)

Context:

- Math 2270, Linear Algebra
  - Learning how to write proofs
  - More abstract thinking
- Class of 45 students
- One class-period of group work every Friday

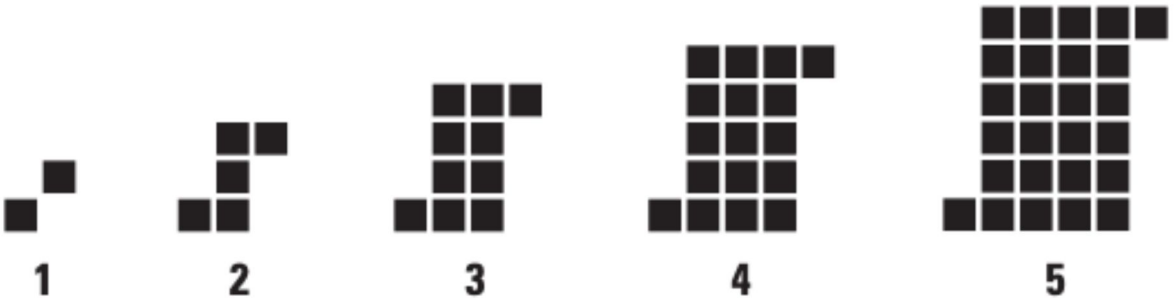
Task done in groups every week.

Following Bleiler et al., 2015 :

Task 1:

- Write a proof
- Accessible mathematical task
  - Slightly open-ended
  - Space to explore before proving

(a) Consider the pattern below. How many square tiles would there be in the 8th step of this pattern?



(b) Write an expression for the total number of square tiles in the figure at an arbitrary step n of the pattern.

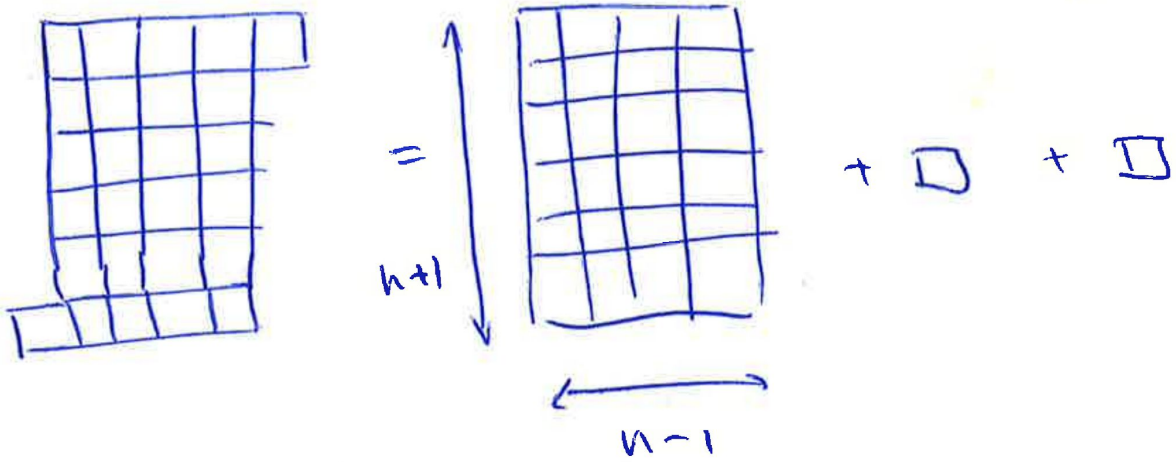
(c) Prove that your expression is a valid representation of the number of tiles at step n of the pattern. You may use drawings, words, numbers and/or symbols for your proof.

Task 2:

- Read compilation of examples of task 1
- Decide if they are convinced by each example
- Write a proof rubric

Some examples:

(c) Prove that your expression is a valid representation of the number of tiles at step n of the pattern. You may use drawings, words, numbers and/or symbols for your proof.



(example with step 5)

Some examples:

(c) Prove that your expression is a valid representation of the number of tiles at step n of the pattern. You may use drawings, words, numbers and/or symbols for your proof.

$$\underbrace{\text{n-long row : top and bottom}}_{2n}$$

$$\underbrace{(n-1) \times (n-1) \text{ square in between}}_{(n-1)^2}$$

Total: $2n + (n-1)^2$

Some examples:

(c) Prove that your expression is a valid representation of the number of tiles at step n of the pattern. You may use drawings, words, numbers and/or symbols for your proof.

$a_1 = 0 \cdot 2 + 2$

$a_2 = 1 \cdot 3 + 2$

$a_3 = 2 \cdot 4 + 2$

$a_4 = 3 \cdot 5 + 2$

$a_5 = 4 \cdot 6 + 2$

$\vdots$

$a_8 = 7 \cdot 9 + 2$

So

$a_n = (n-1)(n+1) + 2$

1

Some examples:

(c) Prove that your expression is a valid representation of the number of tiles at step n of the pattern. You may use drawings, words, numbers and/or symbols for your proof.

$$n^2+1 = 2+(n-1)(n+1)$$
$$= 2+n^2-n+n-1$$
$$= 2+n^2-1$$
$$= 1+n^2$$

✓

1

First rubric!

Correctness	General	Replicability	Clear and precise	Steps
• Start with appropriate assumption, • No logical mistakes, • Not assume what you want to prove	• Work for all possible cases and all relevant values, • No missing cases	• Someone else can read it and get to the same conclusion, • Can recreate the same answer through different ways	• Understandable without additional information, • Visual models when applicable, • Explicit description, • Include important details, • Name properties or strategies, • Define variables	• Logical order, • No missing steps, • Sentences • Conclusion

Task 3:

- Use class rubric to write a new proof
  - Linear algebra task
- Modify rubric as needed

Instructions: rubric should become

- Complete
- Easy to use

First proof using the rubric:

1. As we progress through the semester, we will keep updating our proof rubric. The next page is our first version. It is a summary of all the contributions from the Food for Thoughts from week 2. Use this rubric when writing your proof in this first problem.

   (a) Prove the following statement: If the vectors $\vec{v}, \vec{w}$ are linearly dependent, then the set $\{\vec{v}, \vec{w}, \vec{z}\}$ is linearly dependent.

   (b) Edit the rubric on the next page as needed. The goal is that, as a class, we obtain a rubric which is both complete and easy to use. Find at least two contributions to make to the rubric, as a group. You may add items in a column, add a new criterion, merge columns, change the wording, etc.

Updated rubric: Criteria merged, rephrased, added.

Correctness	General	Replicability	Clear and precise
• Starts only with proven assumption(s), • No logical mistakes, • Logical order, • No mathematical mistakes	• Works for all possible cases and all relevant values, • States the results/theorems used, if any, • Includes relevant information only, • Includes counter-examples or examples when needed	• Can easily get to the same conclusion when reading it, • Path does not matter as long as it is correct, • Leads to conclusion	• Names properties and/or strategies, • Defines variables and what values they can take, • Detailed work is shown, • Phrases or explanations (could be short-hand)

Next proof:

1. **Our proof of the week!** Let A be a 6×4 matrix and B a 4×6 matrix. Show that the 6×6 matrix AB cannot be invertible.

   Use our proof rubric, which is attached as the last page of this FFT. Do not forget to update the rubric as needed.

(Newly) updated rubric: in progress!

Repeat!

Task n:

- Use (updated) class rubric to write a new proof
- Modify rubric as needed

Observations about the rubric (so far):

- Important points came up naturally in the rubric
- Comparing with Kelly's class' rubric, many similarities!
- Included various types of problems to get different perspectives

But...

- Maybe too much (unnecessary) detail?
- Updates not always going as I would hope...

Observations about the students' behavior (so far):

- Question each other's reasoning often
- Feel comfortable bringing up their own ideas/suggestions during class

But...

- Still rely on the teacher "Is this a valid proof?" for now
- Editing the rubric feels artificial for some students
- Do not **use** the rubric that much when writing proofs

Challenges for the teacher

Time !

- Re-discovering results takes more time
- Wait until every student gets to the result?
- Constraints from the curriculum

- Creating new activities (teacher's time)

Finding the right amount of guidance

- Not reinventing the wheel !
- Notations/vocabulary need to match the mathematics literature

Some topics work better than others

Against students expectations of what the teacher's role should be (Marks, 2009).

Too much organizational/procedural autonomy can lead to less critical thinking (Stefanou et al., 2004).

Need to **teach** students to become owners of their learning. Not only give space for it.

Hard to assess within the classroom setting

*"Many aspects of student ownership of learning can be inferred from a **variety of academic performances.**"*

(Conley & French, 2014)

Some small actions...

Examples to teach metacognition (Paris & Paris, 2001):

- Role modeling
- Open discussion
- Reciprocal teaching
- Direct instructions

Some small actions…

Group work: Being confronted with others' understanding

- Increases ownership of mathematics
- Leads to self-reflection

Different groups on different tasks:

- Less time consuming
- Students become expert on a topic

Some small actions…

Offer challenging problems, without pressure (graded for completion, for example)

Offer feedback without grades (to help model internal feedback)

Give open-ended assignments (projects, exploration of a topic)

Some small actions…

(Guided) self-assessment of learning habits/strategies paired with other assignments

Questioning the student's reasoning **after** they thought about it thoroughly

- ask about "how" (in addition to "why")

Summary

The five components of ownership of learning

- Ownership of mathematics increases motivation
- Ownership of learning impacts achievement,
- Supports diversity,
- Can be challenging to teach or develop in classes,
- Specific strategies can help!

Some reflections to take with you…

As a teacher… (Marks, 2009)

- When/how do I intervene socially?
- When/how to I intervene mathematically?
- What do my students lose when I am controlling the mathematics?
- What are my students' assumptions about the classroom?

Thank you for your attention!

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