MATH 1030-004, Exam 2 Solution **Spring 2014**

1. **(16 pts)** You are in the process of buying a house, and you need a mortgage loan of \$200,000. You got approved for a 15-year fixed rate loan at an APR = 4.5%.

   (a) What is the monthly payment?

$$
\begin{aligned}
PMT &= \frac{P \cdot \left(\frac{APR}{n}\right)}{1 - \left(1 + \frac{APR}{n}\right)^{-nY}} \\
&= \frac{200,000 \cdot \left(\frac{0.045}{12}\right)}{1 - \left(1 + \frac{0.045}{12}\right)^{-12 \cdot 15}} = \$1,530.
\end{aligned}
$$

   (b) How much, in total, will you pay for this house after 15 years?

$$\text{Total Paid } = \$1,530 \cdot 12 \cdot 15 = \$275,400.$$

   (c) How much will go toward interest, and how much toward the principal?

   $\$200,000$ goes toward principal, and $\$275,400 - \$200,000 = \$75,400$ toward interest.

2. **(16 pts)** A rental car company charges a basic fee plus a price per mile. One customer drove 60 miles and paid \$70, while another drove 420 miles and paid \$106.

(a) Write an equation for the cost of renting a car.

We are given two points here, $p_1 = (60, 70)$ and $p_2 = (420, 106)$. So

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{106 - 70}{420 - 60} = 0.1.$$

Now we use p_1 to compute b in $y = b + 0.1x$

$$\begin{aligned} 70 &= b + 0.1 \cdot 60 \\ 70 &= b + 6 \\ b &= 70 - 6 = 64. \end{aligned}$$

So the equation is $y = 64 + 0.1x$.

(b) What is the basic fee (in dollars) this company charges?

It is the initial, at $x = 0$ miles, cost, which is $b = 64$.

(c) How much would you pay if you drove a car for 2,000 miles?

$$y = 64 + 0.1 \cdot 2,000 = \$264.$$

3. **(16 pts)** The number of bacteria in a bottle doubles every 15 minutes.

(a) How long will it take for the number of bacteria to grow to five times its initial count?

$$\begin{aligned} Q &= Q_0 \cdot 2^{t/T_d} \\ 5 &= 1 \cdot 2^{t/15} \\ \log_{10} 5 &= \log_{10} 2^{t/15} \\ \log_{10} 5 &= \frac{t}{15} \cdot \log_{10} 2 \\ t &= 15 \cdot \frac{\log_{10} 5}{\log_{10} 2} = 34.83 \text{ minutes.} \end{aligned}$$

(b) If, initially, there were only 7 bacteria in the bottle, how many will there be in 4 hours?

$$Q = 7 \cdot 2^{(4 \cdot 60)/15} = 7 \cdot 2^{240/15} = 458,752.$$

4. **(16 pts)** Andrea has $\$17,500$ in her account. Every week she spends $\$140$, and does not add money to her account.

(a) Write a linear equation for this situation.

The rate of change is $\$140$ per week; in fact, Andrea's account is decaying by $\$140$ every week. So the slope is $m = -140$. The initial value is $b = 17,500$. Therefore, the equation becomes

$$y = 17,500 - 140x.$$

(b) In how many days is Andrea going to spend all of the money in her account?

$$\begin{aligned} 0 &= 17,500 - 140x \\ 140x &= 17,500 \\ x &= \frac{17,500}{140} = 125 \text{ weeks.} \end{aligned}$$

$$125 \text{ weeks} \cdot \frac{7 \text{ days}}{1 \text{ week}} = 875 \text{ days}$$

(c) How much will she have left in her account after 1 year? *(1 year = 52 weeks.)*

$$y = 17,500 - 140 \cdot 52 = \$10,220.$$

5. **(20 pts)** 800 grams of plutonium in a bone specimen decays at a rate of 0.69% per year.

(a) Find the exact half-life of plutonium.

$$T_h = -\frac{\log_{10} 2}{\log_{10}(1 - 0.0069)} = 100.11 \text{ years.}$$

(b) Write an exponential equation for the given situation using the decay rate.

$$Q = 800 \cdot (1 - 0.0069)^t = 0.9931^t.$$

(c) Write an exponential equation for the given situation using the half-life.

$$Q = 800 \cdot 0.5^{t/100.11}$$

(d) How much plutonium will remain after 350 years?
(Hint: there are 800 grams of plutonium in a bone specimen initially.)

$$Q = 800 \cdot 0.9931^{350} = 70 \text{ grams.}$$

Or

$$Q = 800 \cdot 0.5^{350/100.11} = 70 \text{ grams.}$$

6. **(16 pts)** Stan, a used-car salesperson, earns a base salary of \$10,000 plus a commission of \$750 for every car he sells.

   (a) Write a linear equation describing this situation.

   Here the rate of change is \$750 per car sold, so $m = 750$, while the initial pay is \$10,000, so $b = 10,000$. Therefore,
   $$y = 10,000 + 750x.$$

   (b) What would Stan's income be if he sold 26 cars?
   $$y = 10,000 + 750 \cdot 26 = \$29,500.$$

   (c) How many cars does Stan need to sell to earn a \$40,000 income?
   $$\begin{aligned} 40,000 &= 10,000 + 750x \\ 750x &= 40,000 - 10,000 = 30,000 \\ x &= \frac{30,000}{750} = 40. \end{aligned}$$
   He needs to sell 40 cars to earn a \$40,000 income.

7. **(Extra Credit: 10 pts)** Initially, a bacteria culture contained $3,200$ bacteria. The count increased to $67,000$ bacteria 2.3 hours later. What is the doubling time for this bacterium?
$$\begin{aligned} Q &= Q_0 \cdot 2^{t/T_d} \\ 67,000 &= 3,200 \cdot 2^{2.3/T_d} \\ 20.94 &= 2^{2.3/T_d} \\ \log_{10} 20.94 &= \log_{10} 2^{2.3/T_d} \\ \frac{2.3}{T_d} &= \frac{\log_{10} 20.94}{\log_{10} 2} = 4.3882 \\ T_d &= \frac{2.3}{4.3882} = 0.524 \text{ hours} = 31.45 \text{ minutes}. \end{aligned}$$