1. (16 pts) You are in the process of buying a house, and you need a mortgage loan of $200,000. You got approved for a 15-year fixed rate loan at an APR = 4.5%.

   (a) What is the monthly payment?

   \[
   PMT = \frac{P \cdot \left( \frac{APR}{n} \right)}{1 - \left( 1 + \frac{APR}{n} \right)^{-nY}}
   \]

   \[
   = \frac{200,000 \cdot \left( \frac{0.045}{12} \right)}{1 - \left( 1 + \frac{0.045}{12} \right)^{-12 \cdot 15}} = $1,530.
   \]

   (b) How much, in total, will you pay for this house after 15 years?

   \[
   \text{Total Paid} = 1,530 \cdot 12 \cdot 15 = $275,400.
   \]

   (c) How much will go toward interest, and how much toward the principal?

   $200,000 goes toward principal, and $275,400 - $200,000 = $75,400 toward interest.
2. (16 pts) A rental car company charges a basic fee plus a price per mile. One customer drove 60 miles and paid $70, while another drove 420 miles and paid $106.

(a) Write an equation for the cost of renting a car.

We are given two points here, \( p_1 = (60, 70) \) and \( p_2 = (420, 106) \). So

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{106 - 70}{420 - 60} = 0.1.
\]

Now we use \( p_1 \) to compute \( b \) in \( y = b + 0.1x \)

\[
\begin{align*}
70 &= b + 0.1 \cdot 60 \\
70 &= b + 6 \\
b &= 70 - 6 = 64.
\end{align*}
\]

So the equation is \( y = 64 + 0.1x \).

(b) What is the basic fee (in dollars) this company charges?

It is the initial, at \( x = 0 \) miles, cost, which is \( b = 64 \).

(c) How much would you pay if you drove a car for 2,000 miles?

\[
y = 64 + 0.1 \cdot 2000 = 264.
\]

3. (16 pts) The number of bacteria in a bottle doubles every 15 minutes.

(a) How long will it take for the number of bacteria to grow to five times its initial count?

\[
Q = Q_0 \cdot 2^{t/T_d}
\]

\[
5 = 1 \cdot 2^{t/15}
\]

\[
\log_{10} 5 = \log_{10} 2^{t/15}
\]

\[
\log_{10} 5 = \frac{t}{15} \cdot \log_{10} 2
\]

\[
t = 15 \cdot \frac{\log_{10} 5}{\log_{10} 2} = 34.83 \text{ minutes.}
\]

(b) If, initially, there were only 7 bacteria in the bottle, how many will there be in 4 hours?

\[
Q = 7 \cdot 2^{(4 \cdot 60)/15} = 7 \cdot 2^{240/15} = 458,752.
\]
4. (16 pts) Andrea has $17,500 in her account. Every week she spends $140, and does not add money to her account.

(a) Write a linear equation for this situation.

The rate of change is $140 per week; in fact, Andrea’s account is decaying by $140 every week. So the slope is $m = -140$. The initial value is $b = 17,500$. Therefore, the equation becomes

$$y = 17,500 - 140x.$$ 

(b) In how many days is Andrea going to spend all of the money in her account?

$$0 = 17,500 - 140x$$

$$140x = 17,500$$

$$x = \frac{17,500}{140} = 125 \text{ weeks.}$$

$$125 \text{ weeks} \cdot \frac{7 \text{ days}}{1 \text{ week}} = 875 \text{ days}$$

(c) How much will she have left in her account after 1 year? (1 year = 52 weeks.)

$$y = 17,500 - 140 \cdot 52 = 10,220.$$
5. (20 pts) 800 grams of plutonium in a bone specimen decays at a rate of 0.69% per year.

(a) Find the exact half-life of plutonium.

\[ T_h = - \frac{\log_{10} 2}{\log_{10} (1 - 0.0069)} = 100.11 \text{ years}. \]

(b) Write an exponential equation for the given situation using the decay rate.

\[ Q = 800 \cdot (1 - 0.0069)^t = 0.9931^t. \]

(c) Write an exponential equation for the given situation using the half-life.

\[ Q = 800 \cdot 0.5^{t/100.11} \]

(d) How much plutonium will remain after 350 years?

(Hint: there are 800 grams of plutonium in a bone specimen initially.)

\[ Q = 800 \cdot 0.9931^{350} = 70 \text{ grams.} \]

Or

\[ Q = 800 \cdot 0.5^{350/100.11} = 70 \text{ grams.} \]
6. **(16 pts)** Stan, a used-car salesperson, earns a base salary of $10,000 plus a commission of $750 for every car he sells.

(a) Write a linear equation describing this situation.

Here the rate of change is $750 per car sold, so \( m = 750 \), while the initial pay is $10,000, so \( b = 10,000 \). Therefore,

\[
y = 10,000 + 750x.
\]

(b) What would Stan’s income be if he sold 26 cars?

\[
y = 10,000 + 750 \cdot 26 = $29,500.
\]

(c) How many cars does Stan need to sell to earn a $40,000 income?

\[
40,000 = 10,000 + 750x \\
750x = 40,000 - 10,000 = 30,000 \\
x = \frac{30,000}{750} = 40.
\]

He needs to sell 40 cars to earn a $40,000 income.

7. **(Extra Credit: 10 pts)** Initially, a bacteria culture contained 3,200 bacteria. The count increased to 67,000 bacteria 2.3 hours later. What is the doubling time for this bacterium?

\[
Q = Q_0 \cdot 2^{t/T_d} \\
67,000 = 3,200 \cdot 2^{2.3/T_d} \\
20.94 = 2^{2.3/T_d} \\
\log_{10} 20.94 = \log_{10} 2^{2.3/T_d} \\
\frac{2.3}{T_d} = \frac{\log_{10} 20.94}{\log_{10} 2} = 4.3882 \\
T_d = \frac{2.3}{4.3882} = 0.524 \text{ hours} = 31.45 \text{ minutes}.
\]