Unit 9C

- In 9C, we will investigate the exponential function and some of its many applications in mathematical models.

- This exponential modeling is a quantity that has a constant growth rate.

**Exponential Functions**

- An exponential function grows (or decays) by the same relative amount per unit time.

- For any quantity $Q$ growing exponentially with a fractional growth rate $r$ (which $r$ is positive?) equals:

$$Q = Q_0 \times (1 + r)^t$$

Where: $t =$ time, $Q =$ Value of quantity at time $t$, $Q_0 =$ initial value of quantity ($t = 0$), $r =$ fractional growth rate for the quantity.

Negative values of $r$ correspond to exponential decay. *The units of time used for $t$ and $r$ must be the same.*

**Algebra with Logarithms**

Solve for $x$ in the equation $3^x = 100$.

(Magic Penny) When will you have $10$ million?
Some virus spreads at a rate of 3.5% per day, with a 7% death rate. There are 100 people infected today. Compute the following:

(a.) How many people will be infected in 1 year?

(b.) How many will die of this infection in 1 year?

(c.) How long would it take for it to wipe out the world population of 7 billion people, assuming that those who survived the virus have the same chance of getting it again, and the same chance of dying from it.

**Graphing Exponential Growth Functions...**

- The easiest way to graph an exponential growth function is to use points corresponding to several doubling times.
- We start with the point \((0, Q_0)\) that represents the initial value at \(t = 0\).
- For exponentially growing quantities, we know that one doubling the value of \(Q\) is \(2Q_0\). After 2 doublings (\(Q\) is \(4Q_0\)). And so on...

**Graphing Exponential Decay Functions...**

- The easiest way to graph an exponential decay function is to use points corresponding to several half lives.
- We start with the point \((0, Q_0)\) that represents the initial value at \(t = 0\).
- For exponentially decaying quantities, we know that one half life of value \(Q\) is \(\frac{1}{2} Q_0\). After 2 doublings (\(Q\) is \(\frac{1}{4} Q_0\)). And so on...
Alternative Forms of the Exponential Function...

- Our general equation for the exponential function is \( Q = Q_0 \times (1 + r)^t \)

- This contains a growth rate, not doubling time or half-life [chapter 8].

If we are given the doubling time \( T_{\text{double}} \) use the form

\[
Q = Q_0 \times 2^{t / T_{\text{double}}}
\]

If we are given the half-life \( T_{\text{half}} \) use the form

\[
Q = Q_0 \times (1/2)^{t / T_{\text{half}}}
\]

\[\textbf{Ex}\] The average price of a home in a certain town was $86,000 in 1990, but home prices have been falling by 4% per year.

(a.) Create an exponential function of the form \( Q = Q_0 \times (1 + r)^t \) to model this situation.

(b.) Find the exact half-life for the home price, and use it to model the situation.

\[\textbf{Ex}\] In 1995, the enrollment at a certain university was 2400 students.

(a.) If the enrollment increases by 11% per year, what will the enrollment be in the year 2011?

(b.) Find the exact doubling time, and use it to model the situation; then answer part (b.) using this model.
Suppose that poaching reduces the population of an endangered animal by 12% per year. Further, suppose that when the population of this animal falls below 45, its extinction is inevitable (owing to the lack of reproductive options without severe in-breeding).

If the current population of this animal is 1400, in how many years will it face extinction?

A fossilized bone contains about 62% of its original carbon-14. How old is the bone? (The half-life of carbon-14 is about 5700 years.)