Unit 9C

- **In 9C, we will investigate the exponential function and some of its many applications in mathematical models.**
- **This exponential modeling is a quantity that has a constant <u>growth</u> rate.**

Exponential Functions

- An exponential function grows (or decays) by the same relative amount per unit time.
- For any quantity $\boldsymbol{Q}$ growing exponentially with a fractional growth rate r (which r is positive?) equals:

$$\mathbf{Q = Q_o} \times \mathbf{(1 + r)^t}$$

Where: t = time, Q = Value of quantity at time t, Q_o = initial value of quantity (t = 0), r = fractional growth rate for the quantity.

Negative values of r correspond to exponential decay. *The units of time used for t and r must be the same.*

Algebra with Logarithms

Solve for x in the equation $3^x = 100$.

(Magic Penny) When will you have $10 million?

Some virus spreads at a rate of 3.5% per day, with a 7% death rate. There are 100 people infected today. compute the following:

(a.) How many people will be infected in 1 year?

(b.) How many will die of this infection in 1 year?

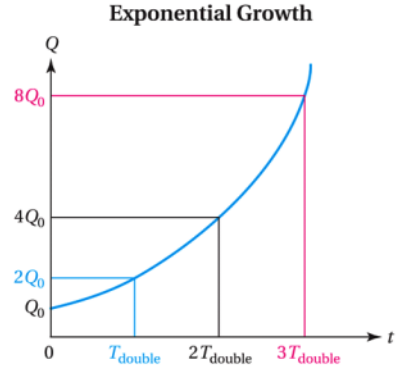
(c.) How long would it take for it to wipe out the world population of 7 billion people, assuming that those who survived the virus have the same chance of getting it again, and the same chance of dying from it.

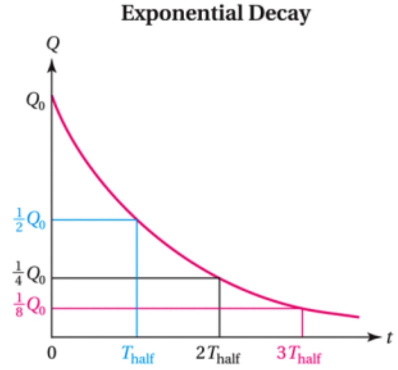
Graphing Exponential Growth Functions...

- The easiest way to graph an exponential growth function is to use points corresponding to several doubling times.
- We start with the point $(0, Q_0)$ that represents the initial value at $t = 0$.
- For exponentially growing quantities, we know that one doubling the value of Q is $2Q_0$. After 2 doublings (Q is $4Q_0$). And so on…

Graphing Exponential Decay Functions...

- The easiest way to graph an exponential decay function is to use points corresponding to several half lives.
- We start with the point $(0, Q_0)$ that represents the initial value at $t = 0$.
- For exponentially decaying quantities, we know that one half life of value Q is ½ Q_0. After 2 doublings (Q is ¼ Q_0).
- And so on…





Alternative Forms of the Exponential Function…

- Our general equation for the exponential function is $\boldsymbol{Q = Q_o \times (1 + r)^t}$
- This contains a growth rate, not doubling time or half-life [chapter 8].

If we are given the doubling time T_{double}, use the form

$$Q = Q_o \times 2^{t\,/\,T_{double}}$$

If we are given the half-life T_{half}, use the form

$$Q = Q_o \times (1/2)^{t\,/\,T_{half}}$$

Ex The average price of a home in a certain town was $86,000 in 1990, but home prices have been falling by 4% per year.

(a.) Create an exponential function of the form $\boldsymbol{Q = Q_o \times (1 + r)^t}$ to model this situation.

(b.) Find the exact half-life for the home price, and use it to model the situation.

Ex In 1995, the enrollment at a certain university was 2400 students.

(a.) If the enrollment increases by 11% per year, what will the enrollment be in the year 2011?

(b.) Find the exact doubling time, and use it to model the situation; then answer part (b.) using this model.

Ex Suppose that poaching reduces the population of an endangered animal by 12% per year. Further, suppose that when the population of this animal falls below 45, its extinction is inevitable (owing to the lack of reproductive options without severe in-breeding).

If the current population of this animal is 1400, in how many years will it face extinction?

Ex A fossilized bone contains about 62% of its original carbon-14. How old is the bone? (The half-life of carbon-14 is about 5700 years.)