

**Section A.1: Functions: the Building Blocks of Mathematical Models**Mathematical models

The purpose of a *mathematical model* is to represent something real (like economic changes) and help us to understand it.

Mathematical models are based on the relationships between quantities that can change. These relationships are described by mathematical tools called *functions*.

**Section A.2: Language and Notation of Functions**Definition of function

A function describes how a dependent variable changes with respect to one or more independent variables. When there are only two variables, we often summarize them as an ordered pair with the independent variable first:

(independent variable, dependent variable)

We say that the dependent variable is a function of the independent variable. If  $x$  is the independent variable and  $y$  is the dependent variable, we write the function as

$$y = f(x)$$

**Ex.1**

Suppose we want to model the variation in temperature over the course of the day based on the data of the following table:

Time	Temperature	Time	Temperature
6:00 a.m.	$50^{\circ}F$	1:00 p.m.	$73^{\circ}F$
7:00 a.m.	$52^{\circ}F$	2:00 p.m.	$73^{\circ}F$
8:00 a.m.	$55^{\circ}F$	3:00 p.m.	$70^{\circ}F$
9:00 a.m.	$58^{\circ}F$	4:00 p.m.	$68^{\circ}F$
10:00 a.m.	$61^{\circ}F$	5:00 p.m.	$65^{\circ}F$
11:00 a.m.	$65^{\circ}F$	6:00 p.m.	$61^{\circ}F$
12:00 a.m.	$70^{\circ}F$		

What is the dependent variable? What is the independent variable?

**Ex.2**

For each situation, express the given function in words. Write the two variables as an ordered pair and write the function with the notation  $y = f(x)$ .

- (1) You are riding in a hot-air balloon. As the balloon rises, the surrounding atmospheric pressure decreases.
- (2) You are on a barge headed south down the Mississippi River. You notice that the width of the river changes as you travel southward with the current.

**Section A.3: Representing Functions**How to represent a function

There are 3 basic ways to represent a function:

- (1) We can represent a function with a *data table*.
- (2) We can draw a picture, or *graph*, of a function.
- (3) We can write a compact mathematical representation of a function in the form of an *equation*.

**The Coordinate Plane**Coordinates in the plane

The points in the coordinate plane are described by two coordinates: the  $x$ -coordinate gives the point's horizontal position relative to the origin, the  $y$ -coordinate gives the point's vertical position relative to the origin.

We use the  $x$ -axis for the independent variable and the  $y$ -axis for the dependent variable. The axes divide the coordinate plane in 4 quadrants.

**Section A.4: Creating Graphs of Functions**Definition of domain and range

The *domain* of a function is the set of values that both make sense and are of interest for the independent variable. The *range* of a function consists of the value of the dependent variable that correspond to the values in the domain.

How to create graph of functions

**Step 1:** Identify the independent and dependent variables of the function.

**Step 2:** Identify the domain and the range of the function. Use this information to choose the scale and labels on the axes. Zoom in on the region of interest to make the graph easier to read.

**Step 3:** Make a graph using the given data. If appropriate, fill in the gaps between data points.

**Step 4:** before accepting any predictions of the model, be sure to evaluate the data and assumptions from which the model was built.

**Ex.3**

Do all the steps for Example 1.

**Ex.4**

Imagine measuring the atmospheric pressure as you rise upward in the hot-air balloon. The table shows typical values you might find for the pressure at the different altitudes, with the pressure given in units of inches of mercury. This pressure unit is used with barometers, which measure pressure by the height of a column of mercury.

Altitude	Pressure
0	30
5,000	25
10,000	22
20,000	16
30,000	10

Use these data to graph a function showing how atmospheric pressure depends on altitude. Use the graph to predict atmospheric pressure at an altitude of 15,000 feet, and discuss the validity of your prediction.