

Section B.1: Writing Large and Small Numbers**Large and small numbers**

Working with large and small numbers is much easier when we write them in a special format called scientific notation.

Scientific notation

Scientific notation is a format in which a number is expressed as a number between 1 and 10 multiplied by a power of 10.

Ex.1

$$\begin{array}{rcl} \text{one billion} & = & 10^9 \text{ (ten to the ninth power)} \\ 6 \text{ billion} & = & 6 \times 10^9 \\ 420 & = & 4.2 \times 10^2 \\ 0.5 & = & 5 \times 10^{-1} \end{array}$$

Ex.2 Numbers in Scientific Notation.

Rewrite each of the following statement using scientific notation.

- (1) The U.S. federal debt is about \$9, 100, 000, 000, 000.
- (2) The diameter of a hydrogen nucleus is about 0.000000000000001 meter.

Approximations with Scientific Notation**Approximations with scientific notation**

We can use scientific notation to approximate answers without a calculator.

Ex.3

Approximate 5795×326 .

Ex.4 Checking Answers with Approximations.

You and a friend are doing a rough calculation of how much garbage New York City residents produce every day. You estimate that, on average, each of the 8 million residents produces 1.8 pounds or 0.0009 ton of garbage each day. Thus the total amount of garbage is

$$8,000,000 \text{ person} \times 0.0009 \text{ ton.}$$

Your friend quickly presses the calculator buttons and tells you that the answer is 225 tons. Without using your calculator, determine whether this answer is reasonable.

Section B.2: Giving Meaning to Numbers**Giving meaning to numbers**

Now that we have a method for writing large and small numbers, we can put numbers in perspective. We will study three techniques to put the number in perspective: through estimation, through comparisons and through scaling.

Perspective through Estimation**Definition of order of magnitude estimate**

An *order of magnitude estimate* specifies only a broad range of values, such as "in the ten thousands" or "in the millions".

Ex.5

We might say that the population of the United States is "on the order of 300 million", by which we mean it is nearer to 300 million than to, say, 200 million or 400 million.

Ex.6

When astronomers say that the number of stars in a galaxy is "of order 100 billion", they mean it could be anywhere within about a factor of 10 of this number, that is, between about 10 billion and 1 trillion. This is a much wider range than the range implied when we say "on the order of 300 million" for the population of the United States, but it is appropriate to the context.

|| Ex.7 Order of Magnitude of Ice cream Spending.

|| Make an order of magnitude estimate of total annual spending on ice cream in the United States.

Perspective through Comparisons**Ex.8**

Consider \$100 billion, which is more or less the wealth of the world's richest individuals. It's easy to say a number like 100 billion, but how big is it?

Ex.9 U.S. versus World Energy Consumption.

Compare the U.S. population to the world population and U.S. energy consumption to world energy consumption. What does this tell you about energy usage by Americans?

Perspective through Scaling

Ways of expressing scales

There are three ways of expressing scales:

- *Verbally*: A scale can be described in words such as "One centimeter represents one kilometer" or, more simply, as "1 cm = 1 km".
- *Geographically*: A marked miniruler can show the scale visually.

- *As a ratio:* We can state the ratio of a scaled size to an actual size. For example, there are 100,000 in a kilometer. Thus, a scale where 1 centimeter represents 1 kilometer can be described as a *scale ratio* of 1 to 100,000.

Ex.10 Scale Ratio.

A city map states, "One inch represents one mile". What is the scale ratio for this map?

Ex.11 Earth and Sun.

The distance from the Earth to the Sun is about 150 million kilometers. The diameter of the Sun is about 1.4 million kilometers and the diameter of the Earth is about 12,760 kilometers. Put this numbers in perspective by using a scale model of the scalar system with a 1 to a 10 billion scale.

Ex.12 Distances to the Stars.

The distance from the Earth to the nearest stars besides the Sun is about 4.3 light-years. On the 1 to 10 billion scale of the last example, how far are these stars from the Earth?

Note: A light-year is the distance that the light can travel in one year:

$$1 \text{ light-year} = 9.5 \times 10^{12} \text{ km.}$$

Ex.13 Timeline.

Human civilization, at least since the time of ancient Egypt, is on the order of 5000 years old. The age of the Earth is on the order of 5 billion years. Suppose we use the length of a football field, or about 100 meters, as a timeline to represent the age of the Earth. If you put the birth of the Earth at the start of the timeline, how far from the line's end would human civilization begin?