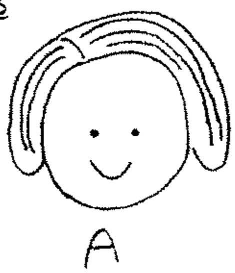
Math Access Notes

Friday June 13!

Public Key Cryptography with Secure Signature

numbering as in Davis' notes

<u>Alice sets up to receive messages</u>



1. Alice picks to large primes
   p, q

2. Alice computes her modulus
   $N := pq$

3a. She chooses the encryption power e relatively prime to $(p-1)(q-1)$

$$[\ \gcd(e, (p-1)(q-1) = 1]$$

3b. $\boxed{\text{The modulus } N \text{ and power } e \text{ are Alice's public key}}$
Anyone wishing to send Alice a "message", i.e a residue $0 \le x < N$, first encrypts it using the function

$$E(x) :\equiv x^e \mod N$$

7. Alice, because she knows number theory and p and q, can find her decryption power d. She solves the multiplicative inverse equation

$$ed \equiv 1 \mod (p-1)(q-1)$$

She will decrypt messages with the function

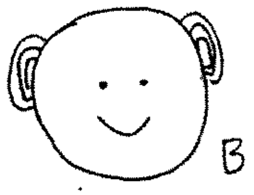
$$D(y) :\equiv y^d \mod N$$

$$\text{since } D(E(x)) \equiv D(x^e) \equiv (x^e)^d = x^{ed} = x^{1 + k(p-1)(q-1)}$$

$$\equiv x \mod N \quad \text{by Fermat-Euler's Thm}$$

the sender's original message!
(assuming x was one of the residue numbers, $0 \le x \le N-1$, and that a residue value is chosen for $D(E(x))$.)

Bob sends a message



4a. Bob wishes to send a message to Alice. He converts it into numbers x, using a conversion key like Davis'.

4b. Secure signature: Bob has created his own public key: e_B, N_B

and private key: d_B

He thinks of a sensible "signature", makes it numeric, s_B, and decrypts it using is private key,

$$D_B(s_B)$$

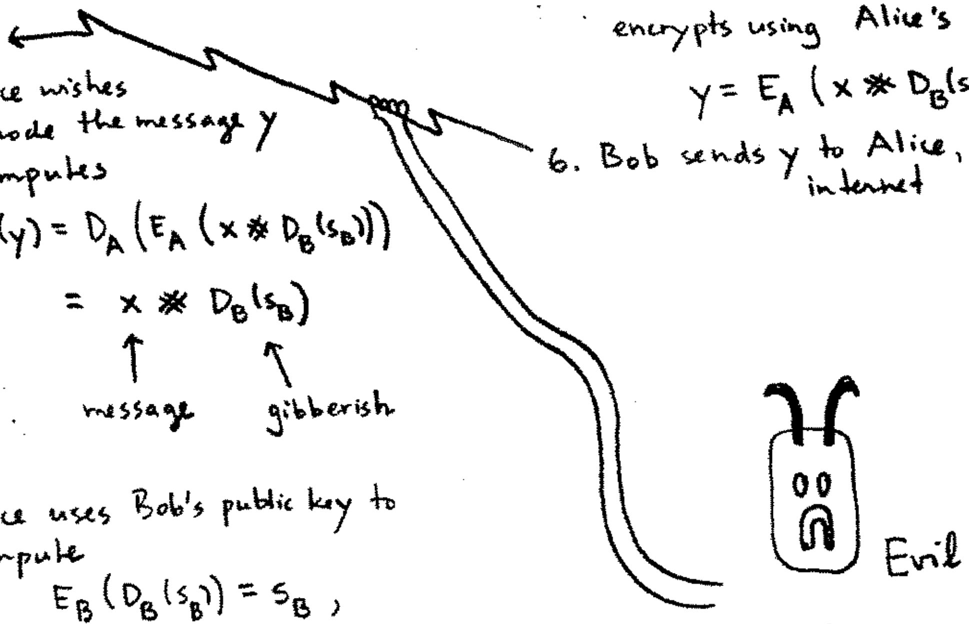
5. Bob appends x to $D_B(s_B)$, creating

$$x * D_B(s_B)$$

(breaks this into blocks $< N_A$) and encrypts using Alice's public key

$$y = E_A(x * D_B(s_B))$$

6. Bob sends y to Alice, e.g. over the internet



8. Alice wishes to decode the message y. She computes

$$D_A(y) = D_A(E_A(x * D_B(s_B)))$$
$$= \underset{\text{message}}{x} * \underset{\text{gibberish}}{D_B(s_B)}$$

9. Alice uses Bob's public key to compute

$$E_B(D_B(s_B)) = s_B,$$

Bobs signature!

[Only Bob could make $D_B(s_B)$!]

Doesn't know D_A so can't get to $x * D_B(s_B)$;

so can't read the message and can't forge messages to Alice which look like they came from Bob.