Historical Fractals

Georg Ferdinand Ludwig Philipp Cantor,
1845(Russia) - 1918(Germany)

Georg Cantor is best known for the Cantor set. To construct the Cantor set, start with the unit interval, delete the (open) middle $\frac{1}{3}$ of this interval, and then successively delete the open middle thirds from the remaining set, and so on. The points which are never deleted in this process comprise the Cantor Set. A related function is the "Devil's Staircase" which is continuous and has derivative equal to zero on 100% (but not all!) of it's domain - yet it is not constant. Cantor loved set theory and the logical underpinnings of mathematics. Hilbert described Cantor's work as "...the finest product of mathematical genius and one of the supreme achievements of purely intellectual human activity."

http://www-history.mcs.st-and.ac.uk/Mathematicians/Cantor.html
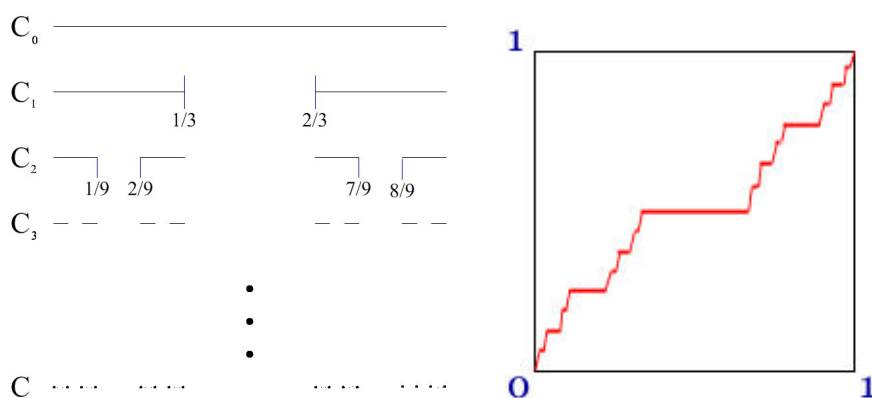

Figure 1: Cantor's Set and the Devil's Staircase

How long is the Cantor set?

Niels Fabian Helge von Koch,
1870 - 1924 (Sweden)

Koch is best known for the Koch snowflake. It is constructed as follows: Begin with the unit interval. Delete the middle open third and replace with two intervals of length $\frac{1}{3}$, forming two sides of an equilateral triangle. Iterate this process, and at each stage replace each edge with 4 sub-edges with $\frac{1}{3}$ the original edge length.

Take the limit of this process, and glue three of the limit pieces together in an equilateral fashion to create the Koch snowflake.

http://www-history.mcs.st-and.ac.uk/Mathematicians/Koch.html
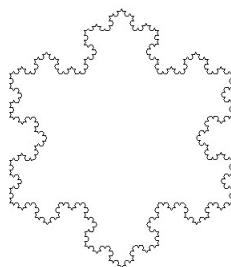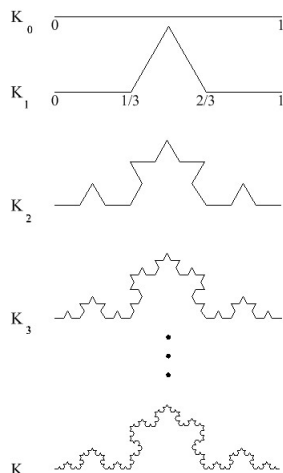

Koch Snowflake

How long is K?

Waclaw Sierpinski
1882 - 1969 (Poland)

You construct the Sierpinski Triangle as follows: Begin with a closed equilateral triangular region with side length 1. At stage 1, subdivide this triangular region into four equilateral subregions, with new edges having half the previous side lengths. Remove the open central sub-triangular region (keeping the edges). Continue this process inductively. The Sierpinski Triangle is the collection of points from the original closed region which never got deleted.

Sierpinski's Triangle

What is its area?
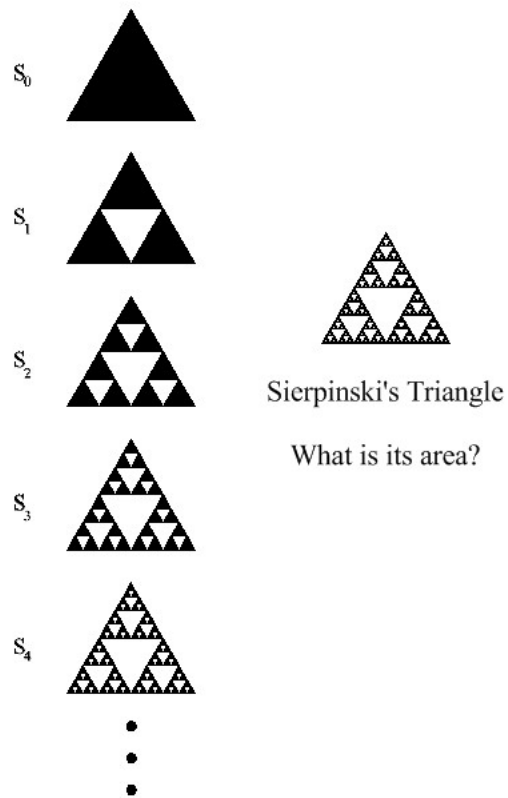
You can also make fractals in 3-dimensional space ...
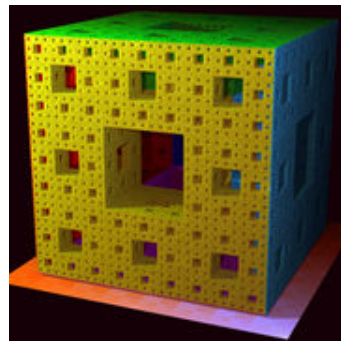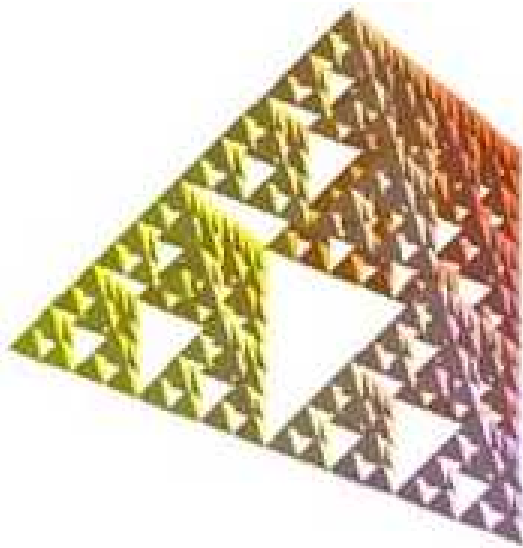
The Menger Sponge

Karl Menger
1902 (Austria) - 1985 (Chicago USA)

How much volume does the Menger Sponge have?

Scaling(Self-Similarity) Dimension

The concept of scaling dimension builds on the ideas we used to understand how area, length, and volume of objects changed under uniform scaling of the background space. There is a much more general notion of dimension, called "Hausdorff dimension", which is also much harder to explain.

Examples:

1. If you scale a line segment by an integer factor of M, you get M^1 pieces which are congruent to the original line segment.

2. If you rescale a square by an integer factor of M, you get M^2 congruent pieces.

Scale by 3 $\longrightarrow$

3^2 congruent pieces
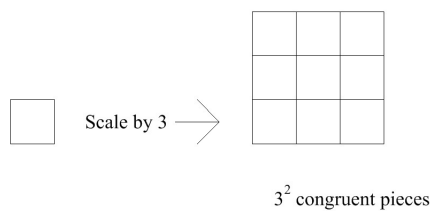
3. If you rescale a cube by a factor of M you get M^3 congruent pieces.

Scale by 3 $\rightarrow$

3^3 congruent pieces
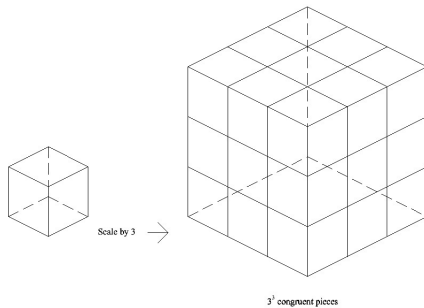
Scaling Dimension: If there is a uniform scaling factor M which transforms an object into N (non-overlapping) pieces congruent to the original object, then the solution d to the equation $N = M^d$ is the scaling dimension. Computationally, $d = \frac{\ln(N)}{\ln(M)}$. Note: M and N could be fractions rather than integers.

Why fractals are called fractals: their dimension is a real number which is usually not an integer!

Exercise 1. Find the scaling dimensions of the Cantor set, the Koch set, and the Sierpinski Triangle. Optional: Find the scaling dimensions of the Sierpinski Pyramid and the Menger Sponge.

Cantor Set

Koch Snowflake

Sierpinski's Triangle