Part II: Harder Transformations

Actually, they are not harder, they are just more general.

$$A\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & c \\ b & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix} = \begin{bmatrix} ax + cy + e \\ cx + dy + f \end{bmatrix}$$

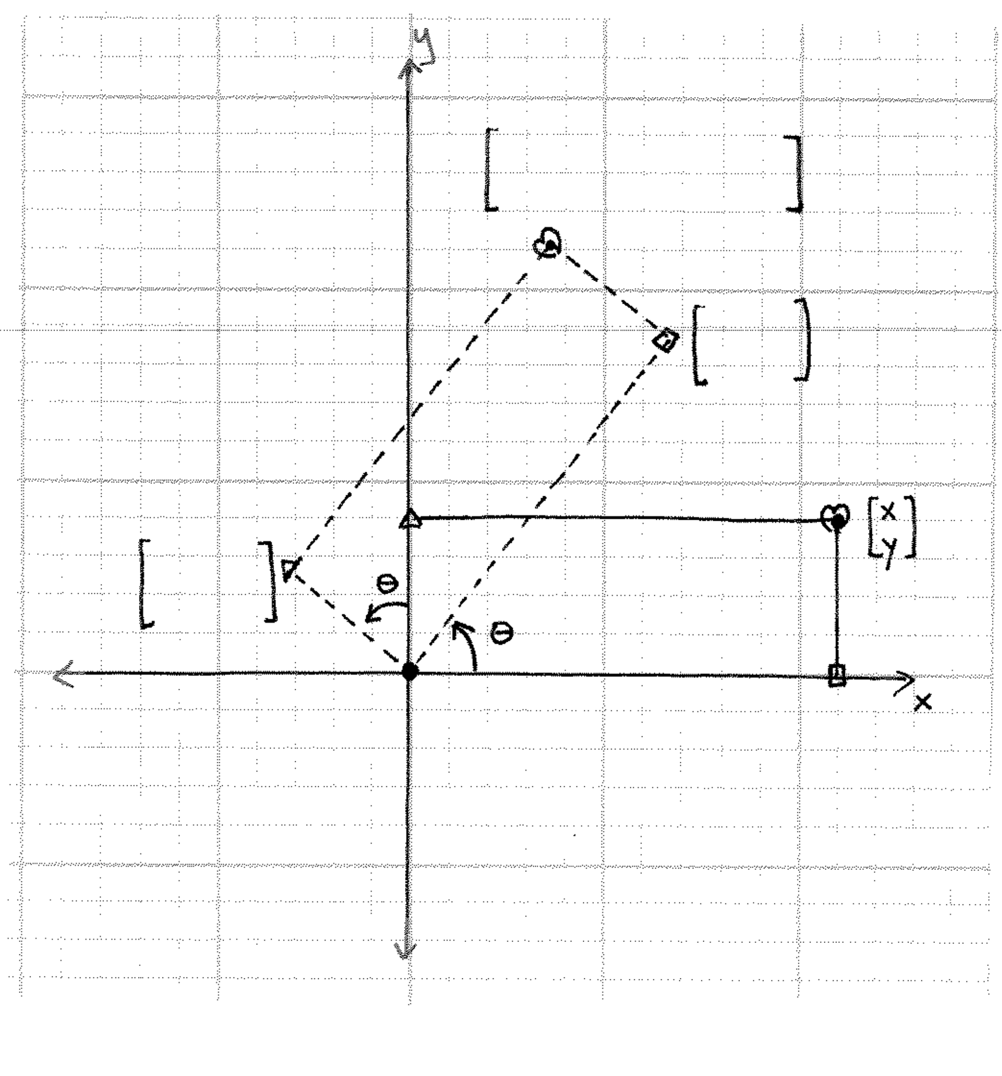
Transformations of this sort are called **affine**. These also include the easy transformations we talked of earlier.

1. Translation: $A\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \quad \\ \quad \end{bmatrix}$
2. Scaling and Translation: $A\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x \\ 5y \end{bmatrix} + \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \quad \\ \quad \end{bmatrix}$
3. Reflection: $A\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -x \\ y \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

etc.

There are also rotations. For example we could rotate by θ radians counterclockwise with the following formula:

$$A\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

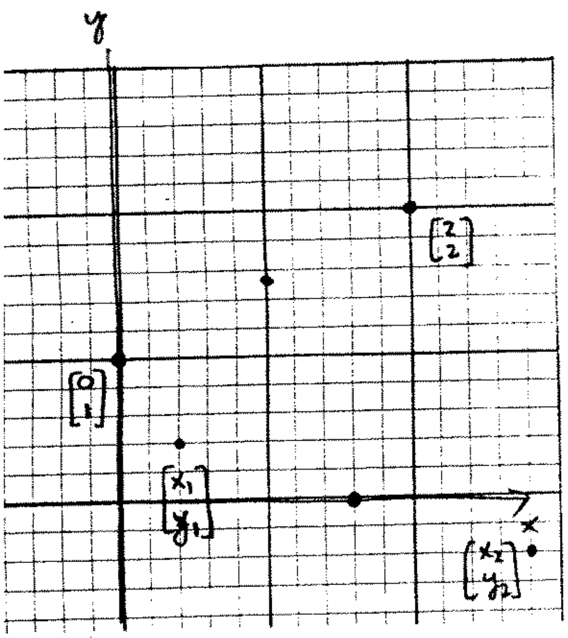


Part III: Geometry of affine transformations

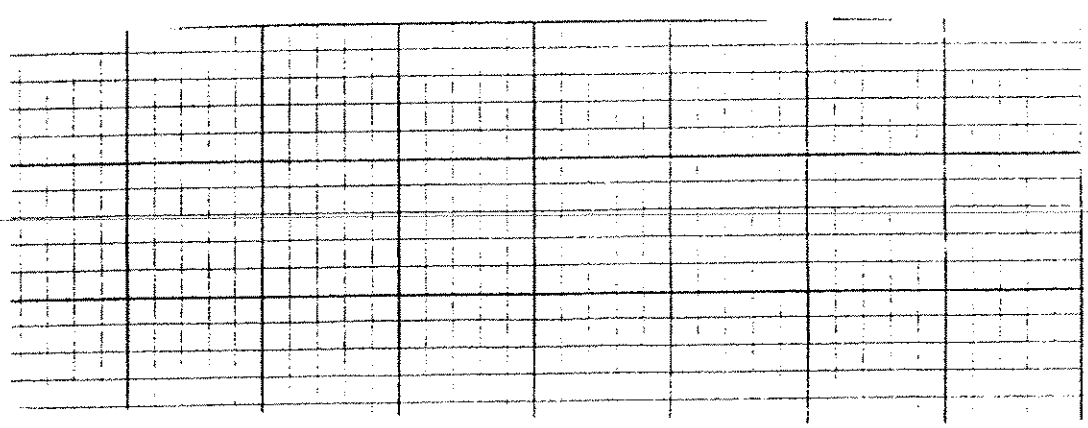
$$A\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & c \\ b & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix}$$

Theorem 1. Let A be an affine map. Then:

1. For any two points P, Q and their midpoint M, the midpoint of $A(P)$ and $A(Q)$ is $A(M)$.



2. Affine maps transform lines into lines.

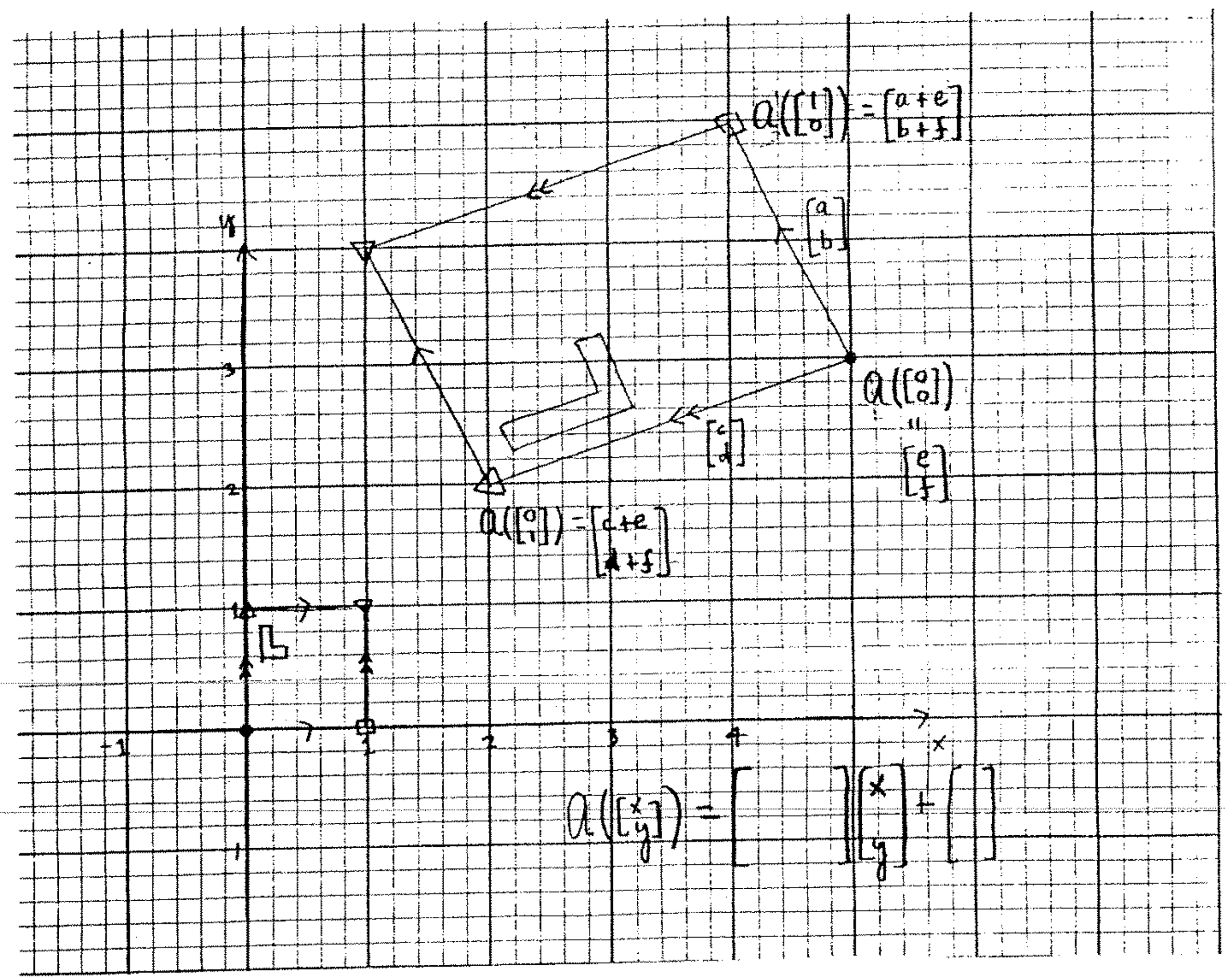


3. Affine maps transform rectangles into parallelograms, so they also transform rectangular grids into parallelogram grids.

Affine transformation template:

$$A\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & c \\ b & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix} = \begin{bmatrix} ax+cy+e \\ cx+dy+f \end{bmatrix}$$

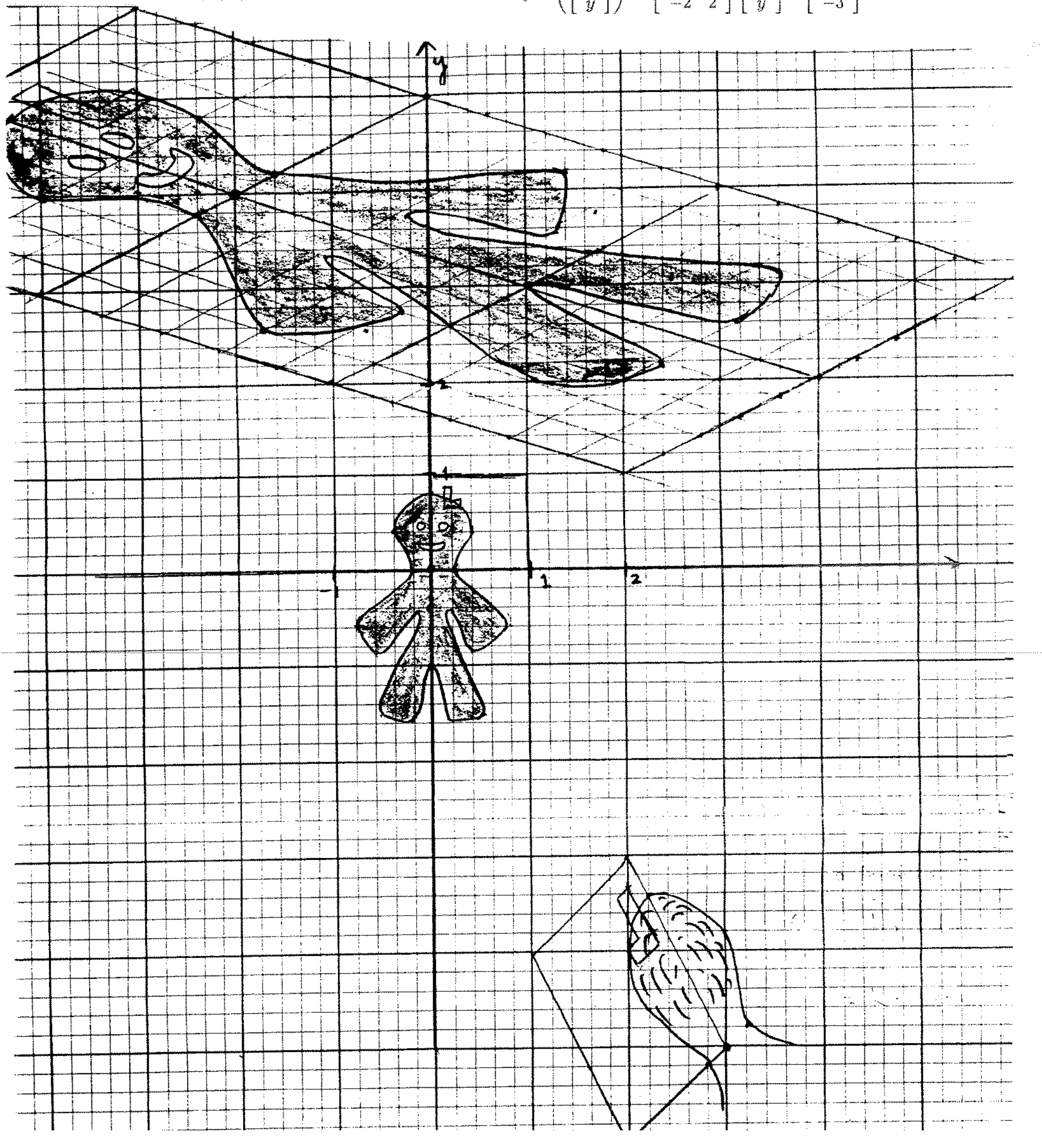
In this formula, $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$ distorts Bob, and $\begin{bmatrix} e \\ f \end{bmatrix}$ translates him.



Bob and L-box transform themsleves:

1. Find formulas for the two affine maps which are shown.

2. Show where the L-box is transformed to by $A\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -4 \\ -3 \end{bmatrix}$.



3. How do affine transformations affect area? How are the area of "Bob" and "A(Bob)" related?

Transformations don't change area, so assume $\begin{bmatrix} e \\ f \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, so $A\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & c \\ b & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$.