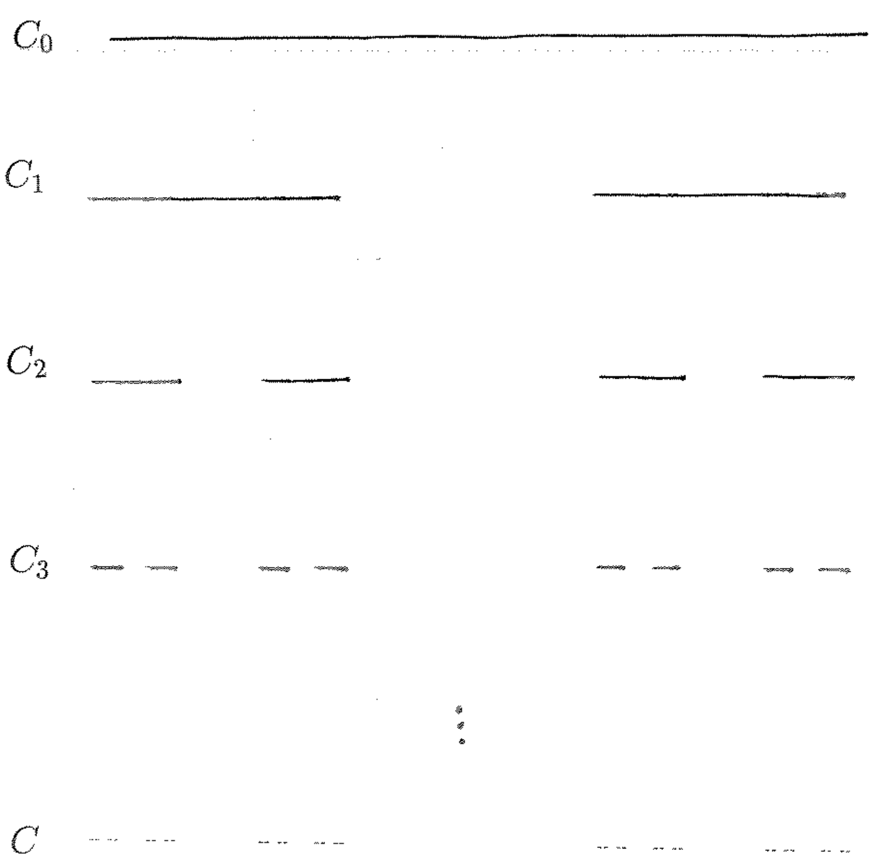
# Part IV: Historical Fractals

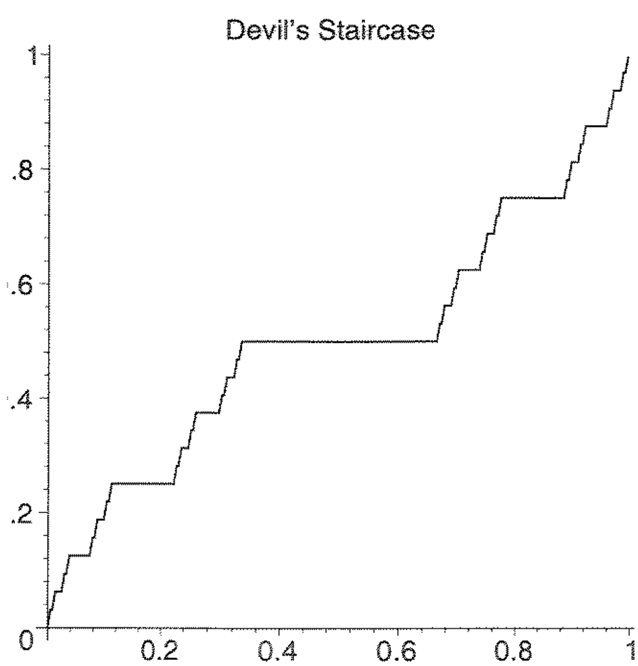
## Georg Ferdinand Ludwig Philipp Cantor 1845(Russia) – 1918(Germany)

Best known for the Cantor set, which is constructed by starting with the unit interval, deleting the (open) middle 1/3 of this interval, and then successively deleting open middle thirds from the remaining set. The points which are never deleted in this process comprise the Cantor Set. A related function is the "Devil's Staircase" which is continuous and has derivative equal to zero on 100% (but not all!) of it's domain – yet is not constant. Cantor, who loved set theory and the logical underpinnings of mathematics, suffered from depression and mental illnesses in his later years.

http://www-history.mcs.st-andrews.ac.uk/history/Mathematicians/Cantor.html




Cantor set

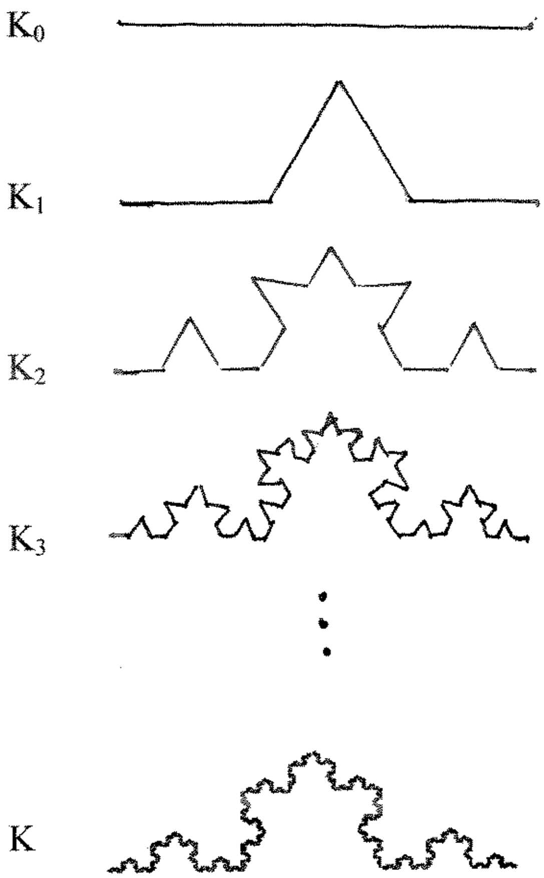


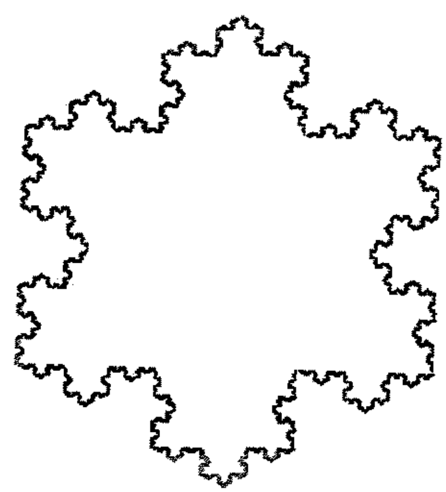

How long is the Cantor Set?

## Niels Fabian Helge von Koch
## 1870-1924 (Sweden)

Best known for the Koch snowflake, made as follows: begin with the unit interval. Delete the middle open third and replace with two intervals of length 1/3, forming two sides of an equilateral triangle. Iterate this process, at each stage replacing each edge with 4 sub-edges having 1/3 the original edge length!

Take the limit of this process (?!), and glue three of the limit pieces together in an equilateral fashion to create the Koch snowflake.



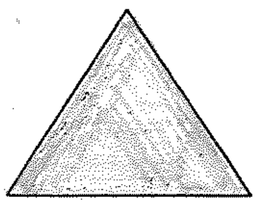
Koch Snowflake

How long is K?

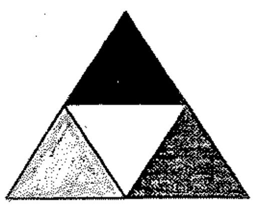
# Waclaw Sierpinski
# 1870-1924 (Poland)

You construct the Sierpinski Triangle as follows:
Begin with the a closed equilateral triangular region,
Say with triangle edge lengths equal to 1.
At stage 1 subdivide this triangular region into four equilateral subregions, with new edges having half the previous side lengths. Remove the open central sub-triangular region (keeping the edges). Continue this process inductively. The Sierpinsiki Triangle is the collection of points from the original closed region which never get deleted.

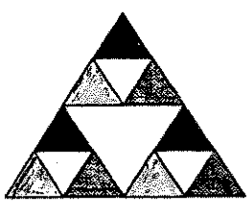
$S_0$



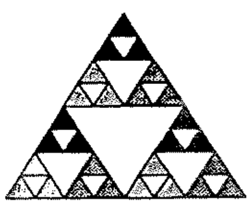
$S_1$



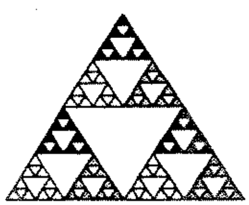
$S_2$



$S_3$



$S_4$



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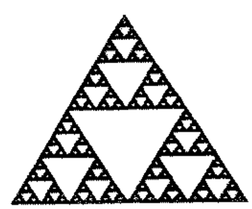
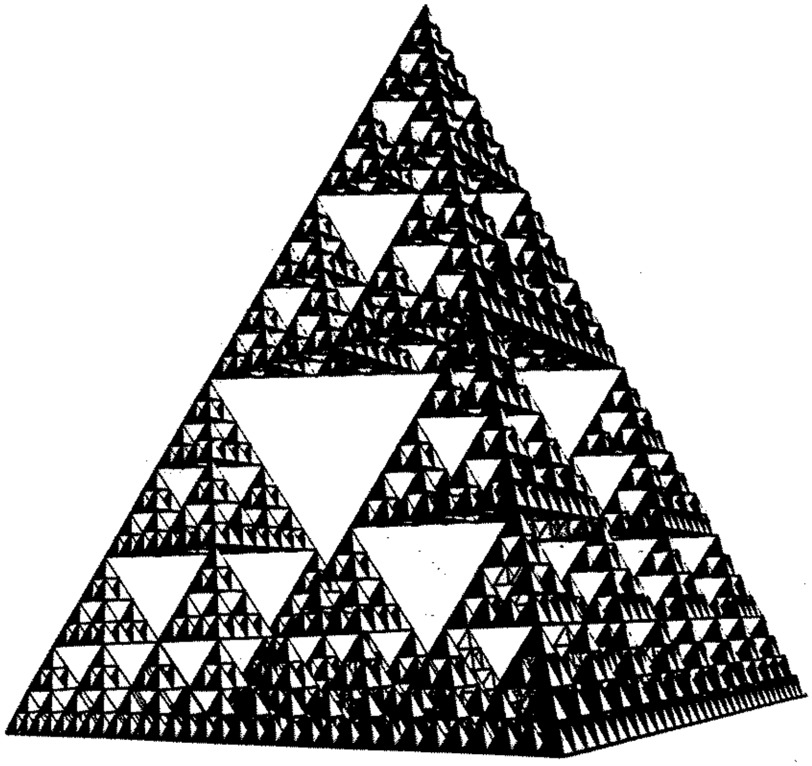


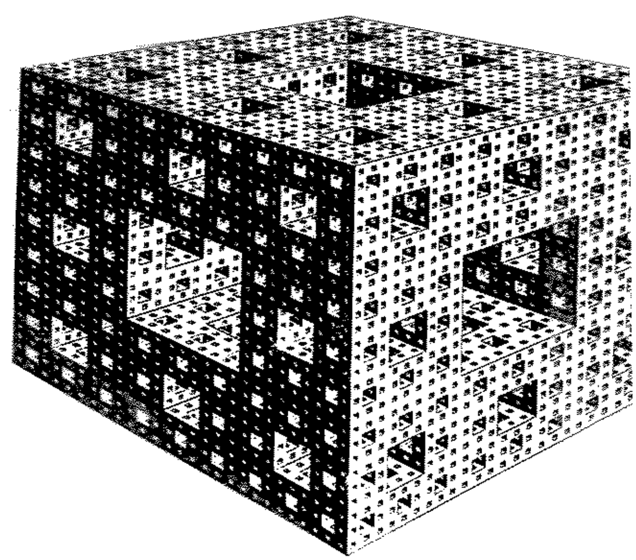
Sierpinski's Triangle
What is its area?

You can also make fractals in 3 dimensional space….



Sierpinski Pyramid

Karl Menger
1902 (Austria) -1985 (Chicago USA)



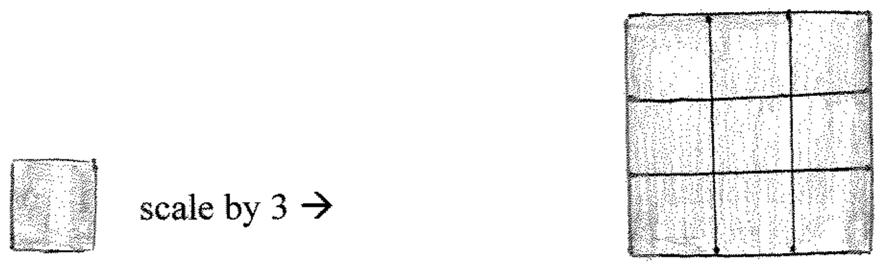
Menger Sponge
(How much volume?)

## Scaling (Self-Similarity) Dimension

The concept scaling dimension builds on the ideas we used to understand how area, length, and volume of objects changed under uniform scaling of the background space. There is a much more general notion of dimension, called "Hausdorff dimension", which is also much harder to explain.

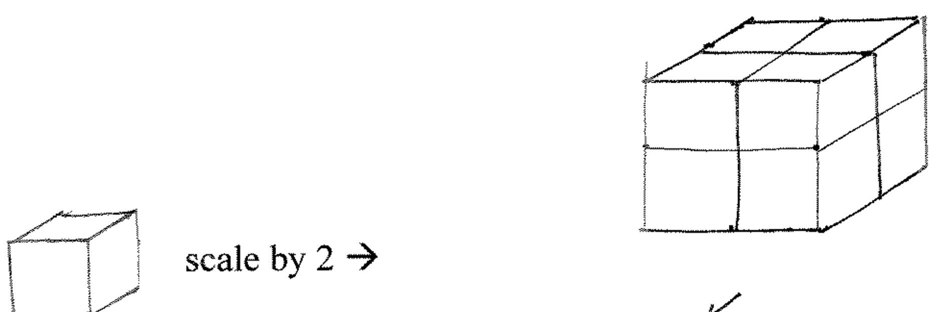
Examples:

(1) If you scale a line segment by an integer factor of M, you get $M^1$ pieces which are congruent to the original line segment.
(2) If you rescale a square by an integer factor of M, you get $M^2$ congruent pieces.




$3^2$ congruent pieces

(3) If you rescale a cube by a factor of M you get $M^3$ congruent pieces.




$2^3$ congruent pieces

<u>Scaling Dimension</u>: If there is a uniform scaling factor M which transforms an object into N (non-overlapping) pieces congruent to the original object, then the solution d to the equation $N=M^d$ , is the scaling dimension. Computationally, $d = \ln(M)/\ln(N)$. Note: M and N could be fractions rather than integers.

Why fractals are called fractals: their dimension is a real number which is usually not an integer!

<u>Exercises:</u> Find the scaling dimensions of the Cantor set, the Koch set K, and the Sierpinski triangle. Optional: find the scaling dimensions of the Sierpinski Pyramid and the Menger Sponge.