Scaling, Self Similarity & Fractals

Lecture notes for Access 2006

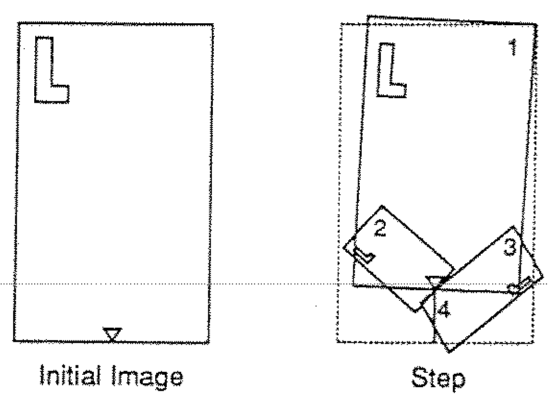


Figure 5.22 : Blueprint of Barnsley's fern.

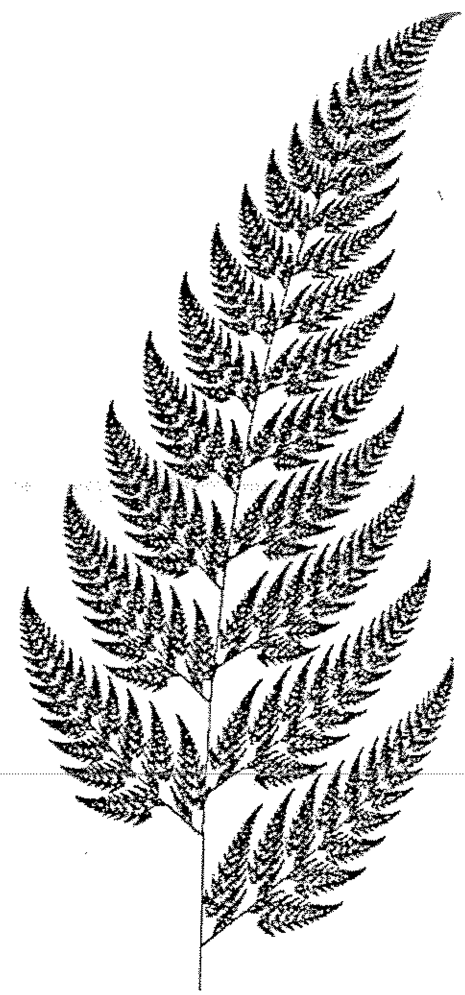
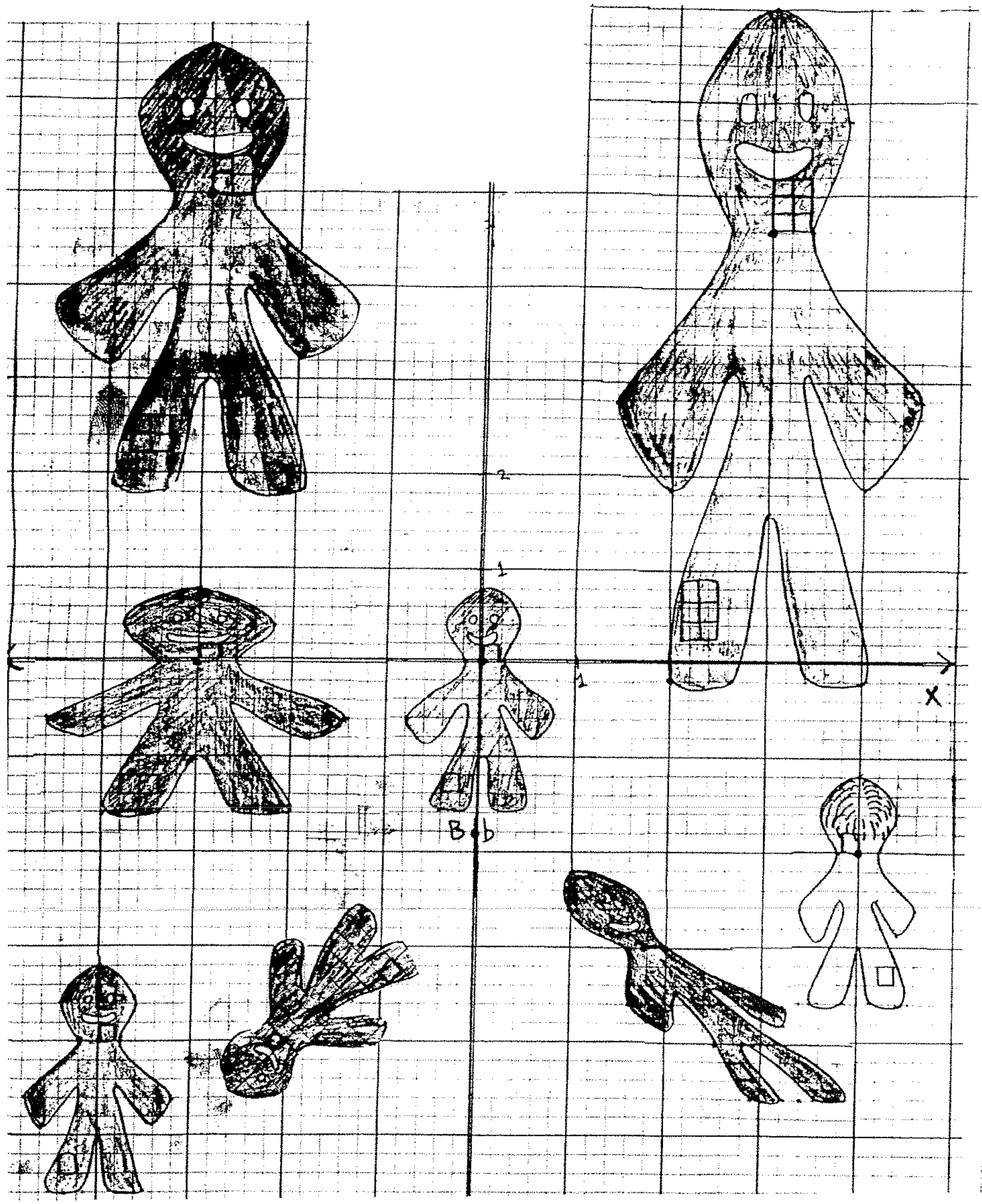


Figure 5.25 : Barnsley's fern generated by an MRCM with only four lens systems.


2
1
1
X
B b

Part I: Classical Scaling

Bob transforms himself. How does he do it?

"Bob" is a collection of points in the plane. If we wish to translate Bob, say 2 units to the right and 1 unit up, we use the transformation function

$$A\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x+2 \\ y+1 \end{bmatrix} := \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

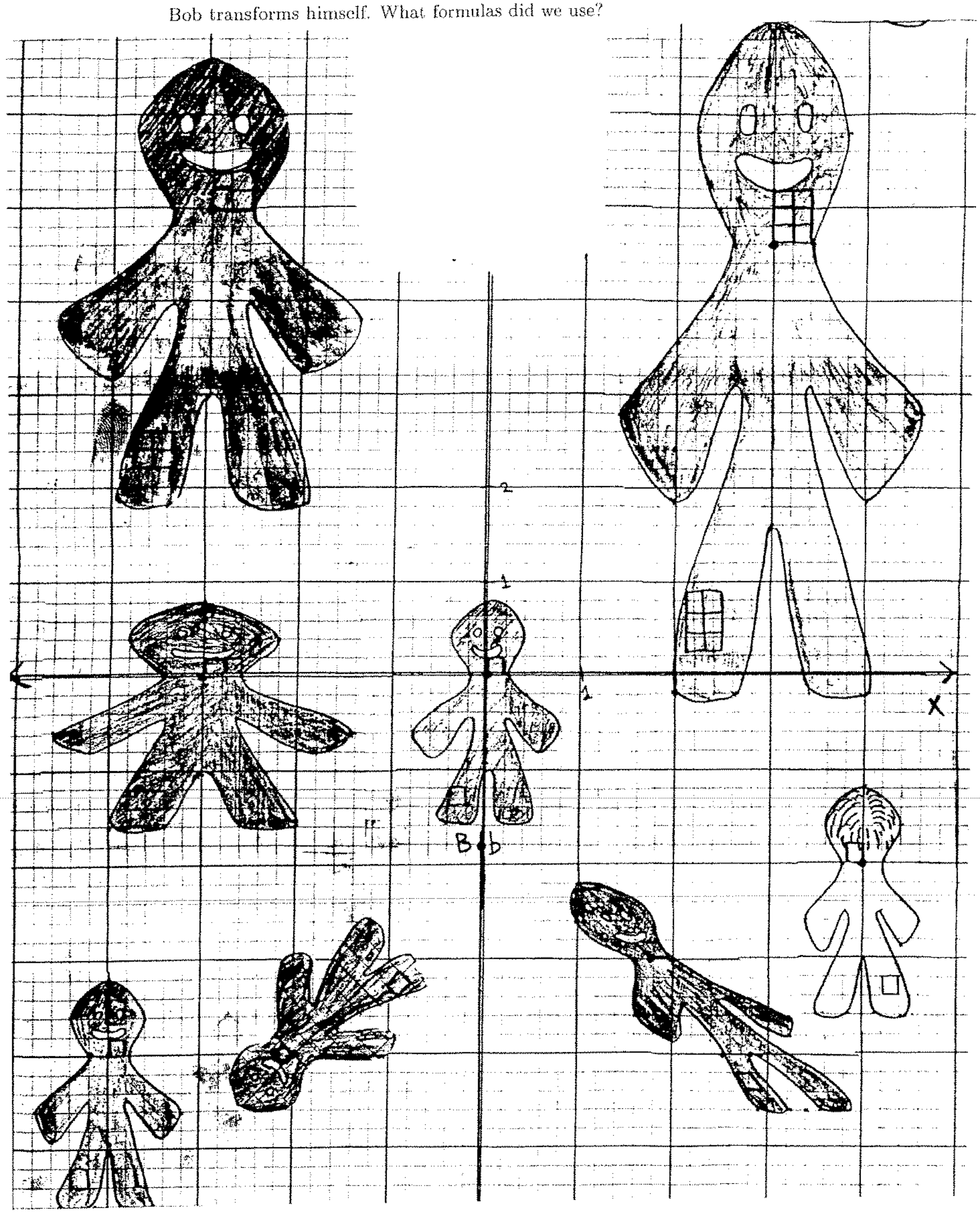
In the above equation, $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ is called the translation vector.

We have the following notation: $A(\text{Bob})$ means the set of $A\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)$'s, where $\begin{bmatrix} x \\ y \end{bmatrix}$ is in Bob.

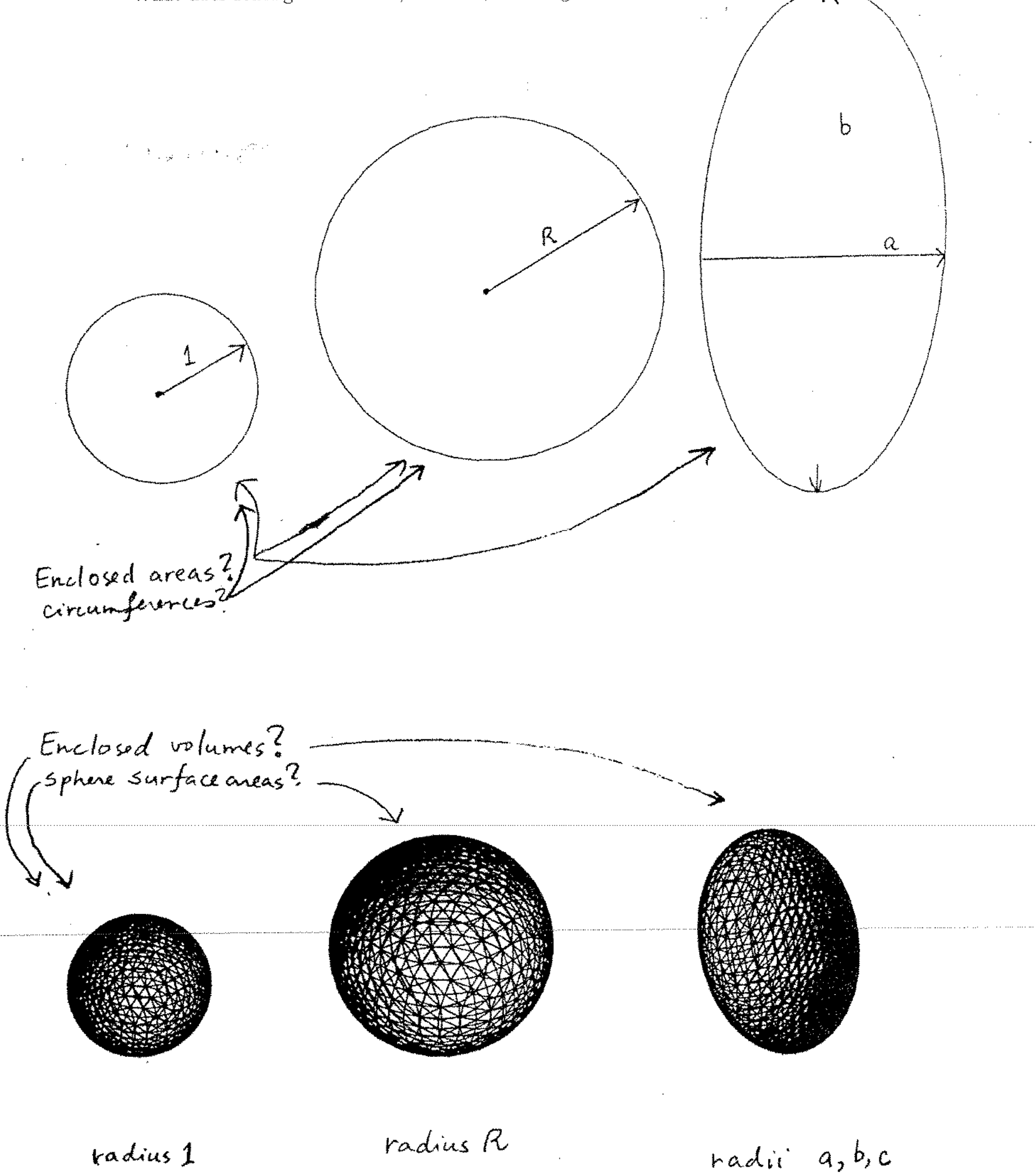
Easy Moves:

1. Translation: $A\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix}$
2. Uniform Scaling, then Translation (say by R): $A\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} Rx \\ Ry \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix}$
3. Nonuniform Scaling, then Translation:
4. Turning Bob over?

Bob transforms himself. What formulas did we use?



What does scaling do to areas, volumes, and lengths?



Classical Scaling:

Why is the Pythagorean Theorem true? ($a^2 + b^2 = c^2$)

Hint: Use area scaling, under uniform scaling: