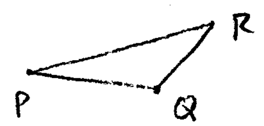
ingredient 1 : Contractions

Let X be a space on which you can measure <u>distance</u>

$$\begin{cases} \text{dist}(P,Q) \geq 0 ; \quad = 0 \text{ only when } P = Q \\ \text{dist}(P,Q) = \text{dist}(Q,P) \\ d(P,R) \leq d(P,Q) + d(Q,R) \end{cases}$$



X is called a metric spa

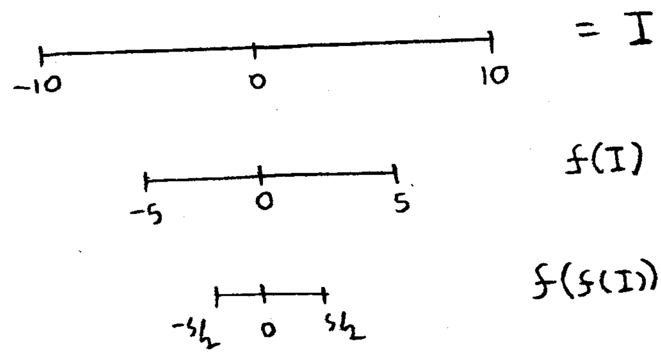
A transformation f of X is a <u>contraction</u> if there is a fraction μ, $0 \leq \mu < 1$ so that

$$d(f(P), f(Q)) \leq \mu\, d(P,Q)$$

<u>Example</u> $f : [-10, 10] \to \mathbb{R}$

$f(x) = \frac{1}{2} x$

Is f a contraction?



Which x's satisfy $f(x) = x$?

these are called <u>fixed points</u>

If we pick any x_0 and set

$x_1 = f(x_0)$
$x_2 = f(x_1)$
$x_3 = f(x_2)$
$\vdots$

What happens to x_n as $n \to \infty$?