

Hutchinson's Theorem 1981

- X a complete metric space

$f_i : X \rightarrow X$ contractions with
contraction constants k_i

Let $F : \begin{array}{c} \text{the space of} \\ \text{subsets of } X \\ \text{(the "Bobs")} \end{array} \longrightarrow \begin{array}{c} \text{the space} \\ \text{of subsets of } X \end{array}$
 $\uparrow$
 a metric space
 with the Hausdorff distance

where $F(\text{Bob}) := f_1(\text{Bob}) \cup f_2(\text{Bob}) \cup \dots \cup f_n(\text{Bob})$

Then F is a contraction, with contraction
constant $k = \max(k_1, k_2, \dots, k_n)$

- So F has a unique fixed point (our fractal!)
and we can get it by picking any
initial subset and iterating F !

[First identify desired fractal
as such a fixed point!]